UNRAVELLING MIXED PROVENANCE OF COASTAL SANDS: THE PO DELTA AND ADJACENT BEACHES OF THE NORTHERN ADRIATIC SEA AS A TEST CASE

Gert Jan Weltje *

Utrecht University, Institute of Earth Sciences, Utrecht, The Netherlands

* Present address: University of Leuven, Faculty of Earth and Environmental Sciences, Department of Geology, Celestijnenlaan 200E, 3001 Leuven-Heverlee, Belgium
E-mail: gertjan.weltje@ees.kuleuven.be

Abstract

Many (if not all) sedimentary basin fills are mixtures of sediments supplied by different sources. A fundamental problem in sedimentary provenance studies is that compositional variation within or among sandstone suites is considered to reflect mixing, but the exact nature of the mixing process is unknown. Observed compositional variation can be cast into the form of a linear mixing model by means of inverse modelling techniques. A new algorithm developed for this purpose is briefly discussed. The inverse modelling approach can help to (1) increase the resolution of provenance models; (2) reduce the problematic effects of varying sampling scales; (3) reduce the extent of prior knowledge required for successful model predictions. The modelling experiment illustrates how spatial patterns of compositional heterogeneity can be used to predict dispersal patterns of sands and the locations of their source areas. The Po-delta front and adjacent beaches of the Northern Adriatic Sea are the ideal natural laboratory for such an experiment. The present situation and late Holocene evolution of this area are well documented, enabling a rigorous testing of model predictions. Compositions of coastal sands, which serve as input for the model, indicate a recycled-orogen or mixed provenance. Results of a series of tests show that the performance of the end-member-modelling algorithm is quite satisfactory. Three out of four modelled end members closely approximate the compositions of sediments supplied by fluvial drainage basins in the area. A fourth end member that is poorly represented in the material studied displays a clear affinity to its actual source composition. The modelled alongshore variation of beach sands, expressed as proportional end-member contributions, is in general agreement with present dispersal patterns and historical records of the area. An overall southwest to southward dispersal suggested by the modelling results is attributable to shifting of Po distributaries during the late Holocene. Recycling of these ancient Po sediments is related to the recent anthropogenic decrease of fluvial sediment supply, and the ensuing widespread coastal retreat in the area studied.

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1. Introduction

The production of sediments and the modification of sediment composition along the pathway between source area and final site of deposition are influenced by a large number of physical, chemical, and biological factors. The sediment-forming process is still not quite understood (Ibbeken & Schleyer, 1991), although considerable advancements in the understanding of sand(stone) provenance have been made in the past decades. Parent lithology, climate and relief are the principal determinants of sand(stone) framework composition (e.g. Dickinson, 1985, 1988; Dutta, 1992). On the scale of global dispersal systems, parent lithology and relief are functions of plate-tectonic setting. The validity of large-scale empirical provenance models, which relate ternary subcompositions of sands and sandstones to plate-tectonic environments (Dickinson, 1985, 1988; Valloni, 1985), has been convincingly demonstrated. However, the provenance discrimination models developed by the Dickinson school are useful for classifying means (i.e. mixtures) of sandstone suites only on the largest scale. Erroneous interpretations may result if local provenance signals in the data have not been suppressed by temporal and spatial averaging of sandstone compositions (Ingersoll, 1990; Ingersoll et al., 1993). This averaging approach has the distinct advantage of robustness, but implies a limited spatial and temporal resolution of current plate-tectonic provenance models. It is therefore not surprising that many sand suites plot in the recycled-orogen and mixed-provenance fields of the ternary provenance diagrams (Dickinson, 1985, 1988).

The analysis of compositional variation of sand(stone) suites on smaller spatial and/or temporal scales cannot rely on generally applicable provenance models. Ingersoll (1990) suggested that small-scale provenance analysis requires a different approach, because "..specific source-rock types, rather than tectonic setting, determine provenance [fields]…" A large number of solutions to problems of local or short-term compositional variation have been proposed. To name a few examples: (1) The compositions of sand(stone)s of known monolithologic parentage has been compared to those of calibrated suites, in order to estimate the combined influence of relief and climate in the source area (Basu, 1985; Grantham & Velbel, 1989; Girty, 1991; Dutta, 1992); (2) Discriminant functions based on compositions of recent sands of known parentage have been used to infer parent lithologies of ancient sands from the same basin (Ingersoll, 1990); (3) A technique which conceptually erodes and mixes known sedimentary parent lithologies has been employed to explain patterns of compositional variation in syntectonic conglomerates (Graham et al., 1986; DeCelles, 1988; Pivnik, 1990; DeCelles et al., 1991). A similar approach was used as part of a comprehensive provenance and mass-balance study of a series of drainage basins along an active margin in Calabria, Italy (Ibbeken & Schleyer, 1991). All of these solutions require a considerable amount of prior knowledge of source area lithology and physiography in order to guarantee successful application. Such detailed information is not generally available for ancient basins.

The short review presented above indicates that provenance studies would greatly benefit from methodological improvements designed to (1) increase the resolution of provenance models; (2) reduce the problematic effects of varying sampling scales; (3) reduce the extent of prior knowledge required for successful model predictions. It will be shown that these problems are closely related, indicating that they can be tackled within the same mathematical framework. A constrained linear model is proposed for describing patterns of compositional variability within and among sand(stone) suites in natural sedimentary systems in terms of mixing of a limited number of end-member assemblages. This model offers a concise description of the system under study and helps to overcome many of the current problems in sedimentary provenance studies.

2. Patterns of compositional variation

The observed compositional variation among and within sand(stone) suites in sedimentary systems may be attributed to a variety of processes. Compositional and textural properties of sediments are affected by selection, mixing, and breakage of grains during transport, as well as chemical-mineralogical transformations during chemical weathering. All of these modifications are selective: the extent to which selection occurs depends on the range of physical and chemical properties of the grain population under investigation (i.e., durability, chemical stability, size, shape, and density).

During transport, each grain behaves differently, depending on its size, shape, and density. Transport of sediments is therefore non-linear with respect to the bulk properties of sediments. It is well known that grain-size distributions are affected by selective transport and deposition (e.g., Komar, 1977; Bardsley, 1978; Dacey & Krumbein, 1979; McLaren, 1981; McLaren & Bowles, 1985; Paola et al., 1992). The same applies to the other bulk physical properties, such as shape and density distributions.

In a sedimentary system, mean transport rates will be different for each size/shape/density class within a grain population. All of the observed spatial variation of bulk physical and textural properties can be attributed to selective transport if no mechanical or chemical weathering occurred in the system being studied, and if the initial characteristics of the sediment supplied to this ideal system did not vary. In such case, all of the observed variation of size, shape, and density distributions must reflect varying proportional contributions of each size/shape/density class to the sediment. Consequently, the chemical and petrographic composition of each size/shape/density class (made up of hydraulically equivalent grains) must be the same at every location. In other words, selective transport alone does not affect the ratios of grain types belonging to a single size/shape/density class.

The concept of framework composition employed by the Gazzi-Dickinson school (Gazzi, 1966; Dickinson, 1970; Ingersoll et al., 1984) is based on ratios of grain types with similar shape and specific gravity (quartzose grains, feldspars, and polymineralic grains). Effects of shape and density selection are minimised by discarding certain classes of grains with deviating properties, such as platy micaceous grains and accessory heavy minerals. In addition, the definition of framework composition generally applies to ratios of principal framework elements within a narrow range of grain sizes. The analysis of compositional modification can be simplified to a considerable extent if it is accepted that compositional variations attributable to size, shape, and density selection can be largely eliminated by quantifying framework compositions of sand-sized sediments according to the criteria defined above. This proposition is supported by studies of Ingersoll et al. (1984) and Zuffa (1985), who demonstrated that framework compositions of different grain-size classes of the same sediments, determined according to the Gazzi-Dickinson conventions, form tight clusters in QFL-space. It may be expected that framework compositions determined according to the Gazzi-Dickinson conventions are equally robust with respect to small variations in shape and specific gravity.

Elimination of selective transport and deposition from our observations leaves three possible mechanisms of compositional variation: mechanical weathering (breakage of grains without chemical or mineralogical modification), chemical weathering (chemical or mineralogical transformation and dissolution), and physical mixing of sediments from multiple sources. If none of these mechanisms contributes to the spatial heterogeneity of bulk sediment properties, its principal framework composition as defined above does not show any spatial variation, and the sediment shed by this source can be represented by a single data point in compositional space. Within this frame of reference, variations of framework composition can be classified in terms of selective and non-selective modifications.

Selective modifications: weathering

The class of principal framework grains can be subdivided into various types of minerals and polymineralic lithic fragments. The chemical and mechanical stability of each grain type determines the rate at which it will dissolve, decompose, or disaggregate under given environmental conditions. Selective modifications, such as chemical and mechanical weathering, treat each type of framework grain differently, and will cause changes to the ratios of framework elements. Mechanical and chemical weathering are therefore non-linear functions of the compositional variables. They can be identified under (natural) laboratory conditions as distinctly curved trends in compositional space. Compositional patterns attributable to chemical weathering can be recognised in sediments that have undergone in situ modification. They are most pronounced in sediments of monolithologic parentage (e.g., soils and saproliths in first-order fluvial drainage basins). Similar effects may be observed in distal parts of sedimentary systems, where sediments from various sources have been homogenised by mixing before they were modified in situ. Two examples of curved compositional trends resulting from progressive non-linear modifications by mechanical weathering (Abbott & Peterson, 1978) and chemical weathering (Johnsson & Meade, 1990) are depicted in the ternary diagrams of Figure 1A and 1B, respectively.

Non-selective modifications: mixing

Physical mixing of sediments from compositionally distinct sources induces strictly linear trends in compositional space, because all principal framework grains of similar size, shape, and density are treated in the same way during the transport process (hydraulic equivalence). For example, all possible mixtures of two compositions can be represented by a straight line segment in a three dimensional QFL-space (Fig. 1C), whereas mixing of sediments from three distinct sources produces a triangular field in QFL-space that is enclosed by the source compositions (Fig. 2A). Linear mixing in sedimentary systems is a primary cause of compositional variation on widely different scales. On the smallest scale, compositional variation of detrital sediments carried by first-order streams reflects contributions of different parent rocks. On a larger scale, compositional variation of sands in large fluvial systems reflects the mixing of sediments delivered by its second-order tributary streams. Patterns of compositional variation in the shallow marine environment can be attributed to mixing and dispersal of sediments from various river systems (third order). A considerable proportion of the observed compositional variation in sandstone suites is thus determined by hierarchical tributary structures, that can be recognised from the uplands in which sediments are produced to the ocean basins in which they are eventually deposited. Examples of this mixing hierarchy are presented by DeCelles & Hertel (1989), Johnsson (1990), and Ingersoll et al. (1993).

The points raised above imply that the perceived effects of a modification process depend largely on the range of physical properties displayed by the grain population under study. By restricting the concept of framework composition to a subpopulation of hydraulically equivalent grains, the effects of selective transport are eliminated and those of physical mixing are linearised. However, this restriction does not affect the non-linearity of other selective processes, such as weathering and diagenesis. In a sedimentary system, selective and non-selective modifications usually go together, as illustrated with the following example. The principal framework compositions of a series of observations arranged along a vertical profile through a soil and saprolith into an unweathered polymineralic parent rock are expected to show a non-linear compositional trend attributable to in situ weathering, represented by the curved line segment in Figure 2B. As soon as this weathering residue is eroded from the bedrock surface, and enters the sedimentary (fluvial) system, its spatial pattern of compositional variation will change due to physical mixing of materials derived from various levels in the original soil profile. The new pattern of compositional variation displayed after transport will consist of a roughly triangular area delimited by the original trend and the straight line between the two extreme compositions (Fig. 2C). The resulting pattern of compositional variation is virtually indistinguishable from that of the ternary mixing system depicted in Figure 2A. Based on these considerations, one might argue that compositional variation in many natural sedimentary systems could be adequately described by a linear mixing model. However, additional requirements for assessing the plausibility of physical mixing must be provided by a coherent spatial distribution of mixing proportions in the area under investigation, and by the geological reasonableness of the source compositions (Imbrie & Van Andel, 1964; Weltje, 1994, 1995).

3. The linear mixing model

A general expression of linear mixing can be formulated as follows (Jöreskog et al., 1976; Menke, 1984; Renner, 1988): Compositional data are generally cast into the form of a matrix X, with n rows representing observations, and p columns representing variables. By definition, all compositional variables are non-negative, and each row of the data matrix sums to a constant c, usually 1, 100, or 10^6 (for measurements recorded as proportions, percentages, or parts per million, respectively).

$$\sum_{j=1}^{p} x_{ij} = c \qquad x_{ij} \geq 0 \tag{1}$$

If compositional variation among a series of measured specimens results from physical mixing, each row of the matrix of compositional data X is a non-negative linear combination M of a fixed number (q) of end-member compositions, represented by the rows of B. In matrix notation, this perfect mixing can be expressed as:

$$X = MB \tag{2}$$

subject to the following non-negativity and constant-sum constraints:

$$\begin{aligned} \sum_{k=1}^{q} m_{ik} &= 1 \qquad m_{ik} \geq 0 \\ \sum_{j=1}^{p} b_{kj} &= c \qquad b_{kj} \geq 0 \end{aligned} \tag{3}$$

Although this representation is acceptable from an algebraic point of view, it does not account for the fact that perfect mixing cannot be demonstrated in practice, due to errors in X. Therefore, the assumption is made that the data matrix X is made up of a systematic part $\hat{X}$, attributable to perfect mixing, and a matrix of error terms E, representing non-systematic contributions to X, such as sampling and measurement errors, and the natural heterogeneity of the materials analysed. In other words, the observed compositional variation is made up of signal and noise. We will assume that the error terms are relatively small and $\hat{X}$ closely resembles X. By definition, the rows of $\hat{X}$, the estimated matrix of perfect mixtures, must consist of non-negative contributions M of q end-member compositions B:

$$\hat{X} = MB = X - E \tag{4}$$

The range of each variable in $\hat{X}$ cannot exceed that of the corresponding variable in the end members B, due to the non-negativity of M. The end-member matrix thus contains the extreme values of each variable. By definition, the perfect mixtures are also compositions, and therefore:

$$\begin{aligned} \sum_{j=1}^{p} \hat{x}_{ij} &= c \qquad \hat{x}_{ij} \geq 0 \\ \sum_{j=1}^{p} e_{ij} &= 0 \end{aligned} \tag{5}$$

The above considerations lead to the following mathematical formulation of the mixing model, which obeys the constraints listed above:

$$X = MB + E \tag{6}$$

In terms of mathematical modelling, Equation 6 is the forward model. If a set of end members has been specified, the composition of any mixture can be produced by straightforward matrix multiplication, as illustrated by Graham et al. (1986) and Ibbeken & Schleyer (1991). However, the fundamental problem in sedimentary provenance studies is the opposite: observed compositional variation is considered to reflect mixing, but the exact nature of the mixing process is unknown. In such cases, the objective is to estimate the mixing parameters from the data by means of inverse modelling techniques. Inverse modelling of parent lithologies can provide important constraints on the nature of the source area; information presently regarded as indispensible for successful forward modelling. For instance, each third-order composition sensu Ingersoll (1990) that is considered suitable for plotting in one of the ternary plate-tectonic provenance diagrams is derived from a data set whose pattern of compositional variation contains a wealth of information about its lower-order parentage. Many of the current problems with respect to model resolution and sampling scale can be reduced significantly, if such information is extracted from the compositional variability displayed by sandstone suites.

4. Inverse modelling strategy

Fitting a mixing model to compositional data is a hazardous task if no a priori information about the number of end members and/or their compositions is available. In such cases, the mixing problem is stated in explicit form, indicating that all of the parameters of the mixing process must be estimated from the data:

- The perfect mixtures $\hat{X}$
- The error terms E
- The number of end members q
- The end-member compositions B
- The mixing coefficients M

A fundamental assumption is that the set of end members from which the observed variability has been generated is linearly independent, meaning that none of the end members can be generated by mixing of other end members (otherwise, the problem cannot be solved). This requirement can be demonstrated by referring to Figures 1C and 2A. Fields in compositional space enclosed by the end-member compositions represent mixing spaces. If the end members are linearly independent, the number of dimensions of the mixing space reflects the number of end members, q. In other words, the number of linearly independent sources, q, is equal to the number of dimensions occupied by the data. This concept of dimensionality is used in the first modelling stage, in order to partition the data matrix into perfect mixtures (signal), and non-systematic contributions (noise), following Equation 4.

A matrix of perfect mixtures can be generated for every value of q (where $2 \leq q \leq p$), by using fundamental concepts of linear algebra (singular value decomposition and constrained weighted least-squares approximation) to ensure that it conforms to the non-negativity constraint of Equation 5. A strategy aimed at constructing an optimal solution to the mixing problem should take into account that the model must be as simple as possible. This implies that the number of end members, q, is determined by the minimum number of dimensions required for a satisfactory approximation of X. The number of end members in the mixing model is thus equal to the approximate dimensionality of the data, which can be estimated without knowledge of the end-member compositions. If desired, the columns of the data matrix can be scaled to equal weights prior to the partitioning into signal and noise, so that each variable is equally important in determining the approximate dimensionality of the data. Without scaling, the weight of each variable largely depends on its standard deviation (Miesch, 1980).

If q has been estimated, a mathematically and geologically feasible mixing model must be constructed which adequately describes the filtered compositional variation. In other words, a matrix of non-negative end-member contributions, M, and a matrix of q realistic end-member compositions, B, must be found, which will yield the perfect mixtures after matrix multiplication (cf. Equation 6). This is a difficult problem, to which no unique solution exists. There may be an infinite number of mixing models that fit a given set of data within tolerable error, and the particular combination of mixing parameters which actually produced the data cannot be reconstructed.

5. The optimal mixing model

A unique unmixing solution can be provided if additional constraints on the model parameters are introduced. A practical strategy is to formulate constraints that will ensure the geological reasonableness of the model. Therefore, additional constraints on M and B are specified as follows. Of primary concern is that the modelled end members enclose the smallest possible mixing space, so that each end member contributes significantly to the observed compositional variation, and its composition can be easily interpreted in geological terms. However, negative values of end-member contributions are not allowed in the model, and a good fit of the model to the approximated data requires that the number and magnitude of negative contributions are as small as possible. In a geometrical sense, the set of end members must enclose as many of the observations as possible, because any observation not enclosed by the end members is distorted in the modelled representation of the data. The apparent contradiction of these two requirements enables the optimal solution for a given (minimal) value of q to be defined as the smallest possible mixing space which encloses a sufficiently large proportion of the data points.

It is also desirable that any strategy for estimating the end-member compositions is robust. Adding or removing a limited number of observations should have a minimal effect on the solution. Because the actual mixing parameters are unknown and cannot be reconstructed from the data alone, it is ultimately up to the modeller to determine which estimates are reasonable under given conditions. The required trade-off between a fully non-negative mixing-proportions matrix on the one hand, and the desire for conservative end-member estimates on the other hand, must also be built in to the algorithm. Therefore, the end-member estimation procedure must be flexible. The simplest way to implement all of the above-mentioned constraints and requirements is by making use of iterative algorithms, which construct an optimal solution from a sensible initial guess of the model parameters.

In the past decades, various strategies for solving the mixing problem have been proposed by Imbrie (1963), Imbrie & Van Andel (1964), Jöreskog et al. (1976), Miesch (1976a, 1980, 1981), Clarke (1978), Full et al. (1981, 1982), Dymond (1981), Dymond et al. (1984), Menke (1984), Leinen & Pisias (1984), Renner (1988), Renner et al. (1989), and Ripley (1990). Although some of these methods may provide a satisfactory answer to the mixing problem, none completely fulfils the general requirements formulated above (i.e. simplicity, mathematical and geological reasonableness, robustness, and flexibility). An inversion scheme which takes into account all of the above-mentioned properties has been developed by the author. A full account of the algebraic details has been published elsewhere (Weltje, 1994, 1995).

A practical application of the inversion algorithm, shown in Figure 3, will serve to illustrate the basic concept of the iterative construction of optimal end-member compositions. The data in the ternary diagram of Figure 3A consist of three source compositions and six mixtures. The number of end members is thus known a priori. Figure 3A also shows the locations of three centres of mass of the data set, that have been calculated using a non-hierarchical cluster algorithm (Full et al., 1982; Bezdek et al., 1984). These centres of mass are the robust initial end-member estimates employed by the iterative procedure. Because a set of end members has been specified, the matrix of mixing proportions corresponding to these end members can be calculated from the exact bilinear relationship of Equation 4. This matrix of mixing proportions is evaluated in order to define improvements to the end members, aimed at reducing the number and magnitude of negative mixing proportions. In the next iteration cycle, a new mixing proportions matrix is generated from the updated matrix of end-member compositions. In a geometrical sense, the mixing space grows in each iteration cycle, until the constraints on an optimal mixing model have been satisfied (Fig. 3B). Modifications to the end-member matrix are based on the collective properties of the data set to minimise the effect of possible outliers on the locations of the end members. In this example, it was assumed that the data were free of error, and the iterative procedure was allowed to run until improvements to the mixing proportions matrix became negligible. Figure 3C shows that the original source compositions are accurately identified and all of the data points are enclosed by the final mixing space.

6. **Application to Northern Adriatic coastal sands**

The current modelling experiment is intended as an analog for the worst case in provenance studies: unravelling the provenance of an ancient basin fill, for which no additional information about hinterland and sediment dispersal is available. In this case, the only input of the inverse model consists of the compositions of modern beach sands. The experiment aims at predicting the locations and compositional signatures of fluvial input. The Po-delta front and neighbouring beaches of the Northern Adriatic Sea (Fig. 4) form an ideal natural laboratory for evaluating the performance of the model, because the present dispersal pattern, the composition of fluvial sands, and the late Holocene evolution of the northern Adriatic coastal plain and Po delta are well documented. This provides an opportunity to assess the usefulness of the inverse modelling approach for the analysis of provenance and sediment dispersal in ancient basins.

As in every case history, there are a number of local peculiarities that may hamper the attempted provenance reconstruction. Beach sands of the northern Adriatic Sea are derived from a few large rivers such as the Po and Adige, and from numerous smaller streams, which drain a lithologically diverse Alpine-Apenninic hinterland composed of siliciclastic sediments, carbonates, igneous rocks, metamorphites, and volcanites (Pigorini, 1968; Nelson, 1970; Gazzi et al., 1973). The predominance of sedimentary rocks in the source areas indicates that most of the detritus delivered to the present Adriatic coast must have been recycled. Zuffa (1987) showed that the average (third-order) composition of the fluvial sands is consistent with a recycled-orogen provenance sensu Dickinson (1985, 1988). Furthermore, beach sands along this wave-dominated coast contain high proportions of mechanically and chemically unstable framework components (lime clasts) that are highly susceptible to selective destruction by wave action (cf. Mack, 1978; Suttner et al., 1981; Sedimentation Seminar, 1988). Another factor that may be expected to obscure the provenance signal is the presently widespread coastal retreat in this area, which re-introduces ancient sands into the modern system (Brambati et al., 1978; Bondesan et al., 1978; Dal Cin, 1983).

The inversion algorithm was applied to a data set collected and analysed by Gazzi et al. (1973). The data set consists of 53 samples of modern beach sands (grain-size range: 1 to 5 φ) from the northern Adriatic coast between Trieste and Pesaro (Italy). The petrographic composition of each sample was estimated by counting 500 points in thin section according to the Gazzi-Dickinson conventions (Gazzi, 1966; Dickinson, 1970; Ingersoll et al., 1984). Sampling stations are depicted in Figure 4. The average inter-sample distance (~ 6 km) and the size of the sampling area (~ 300 km) are comparable with the spatial scale of many provenance studies of ancient sediments. The raw data set was slightly edited for input into the modelling program. A numerical value of 0.1% was assigned to grain types that were reported as occurring in trace amounts in some samples. This value represents the precision of a point count of 500 grains. Gazzi et al. (1973) distinguished 19 classes of grains. Two classes were excluded from the data, because of their deviating shape (micas), or their lack of genetic significance (others). Other grain types that are absent in many samples, and present in small amounts in a limited number of samples only, cannot be adequately represented by the linear mixing model. Such grain types were combined with others based on textural and compositional criteria. A reduction of the number of compositional classes to 10 was considered satisfactory. These modifications are summarised in Table 1. The resulting (53×10) array of beach sands was recalculated to a constant sum of 100%.

A series of approximations to the data was generated, one for each value of q. The minimum number of dimensions (end members) required for a satisfactory approximation of the data was estimated by calculating the coefficients of determination (Fig. 5A). The coefficients of determination represent the proportions of the column variances that can be reproduced by the approximated data. This proportion is equal to the squared correlation coefficient of the input variables and their approximated values (Miesch, 1976, 1980). As shown in Figure 5A, only four variables (quartz, micrite, feldspar, and sparite) can be adequately reproduced by a two-end-member model. The goodness-of-fit of the other six variables increases gradually as the number of end members is increased, indicating that the mixing system is quite noisy. All of the variables (except for chert) can be reasonably well reproduced by a four-end-member model, as indicated by the coefficients of determination that exceed 0.5 (i.e. 50% of the variance is reproduced, equivalent to a correlation coefficient of 0.7). Increasing the number of end members to five only improves the goodness-of-fit of chert to a considerable extent, whereas the number of end members would have to be increased to six to reach a high-precision approximation of the input data. A mixing model with four end members seems to be a reasonable choice in view of the noise in the mixing system, and the desire for a sparse description of the basin fill under study.

In order to investigate if the observations can be adequately represented in four dimensions, the angular deviation and centroid distance of each observation vector were computed, using the coefficient of proportional similarity of Imbrie & Purdy (1962). The angular deviation is defined as the angle between each observation and its predicted composition according to the model. The centroid distance of an observation is defined as the angle between the average composition of the approximated data set and the approximated composition vector. Scatter graphs of angular deviations vs. centroid distance can be used to identify observations for which both values are relatively large (Fig. 5B). Such observations are potential outliers that could influence the modelling results. The scatter graph of Figure 5B shows a random pattern, which indicates that no outliers are present in the four-dimensional approximation of the data.

Four end-member compositions were constructed by means of the iterative procedure described above. A stable solution was obtained after 146 iterations, when improvements to the mixing proportions matrix had become negligible. As expected from the noisiness of the data, negative mixing proportions could not be fully eliminated by the iterative algorithm. Because the average magnitude of negative contributions was small, the goodness of fit of the final mixing model was only slightly reduced by imposing non-negativity constraints on the mixing proportions matrix (Table 2). An a posteriori goodness of fit test indicated that the distribution of centred-logratio residuals (cf. Aitchison, 1986) does not deviate significantly from a multivariate normal distribution, suggesting that only random variation has been discarded by assuming that the data set was generated by mixing of four end members (Renner, 1991; Weltje, 1994). A comparison of the observed and modelled alongshore variation for each grain type is depicted in Figure 6. The major trends of alongshore variation in the beach sands are well reproduced, indicating that a four-end-member representation adequately captures the overall pattern of variation. Extreme values of poorly approximated variables have been smoothed, due to the smaller variance of the model. Systematic deviations from the observed grain content are limited to variables that are poorly reproduced by the model, such as volcanic lithics. Figure 6 shows that the high peak of volcanic lithics between sampling stations 21 and 25 is strongly underestimated in the model.

7. Interpretation of modelling results

The modelling results may be interpreted by plotting the end-member contributions as a function of the spatial sampling coordinates. The spatial pattern of end-member contributions to the Adriatic beach sands is coherent and easily interpretable (Fig. 7). End member 1 contributes a large proportion of material to beach sands in the northern part of the area (stations 1-20), whereas its contributions to the other beach sands are very small. End member 2 contributes a large proportion of material to the beaches located between stations 21 and 25. Contributions of end member 3 are virtually absent in the northern part of the area, but increase dramatically on the beaches between stations 26 and 39. The contribution of end member 3 can be traced to station 43, where it diminishes rapidly towards the south. End member 4 contributes a large proportion of material to the beaches between stations 44 and 53. The compositions of beach sands between stations 40 and 44 can be largely attributed to mixing of end members 3 and 4. The alongshore variation can be conveniently summarised in terms of four sedimentary petrological provinces (*sensu* Edelman, 1933). For future reference, these provinces are labelled A, B, C, and D (from northwest to southeast). The skewed patterns of contributions from end members 2 and 3 could reflect a predominantly south-east-ward alongshore dispersal, although the gradual transition from province C to province D could also indicate a northward dispersal of sands in province D.

The alongshore contributions of end members 1 and 4 show a subsidiary mode, suggesting that a small proportion of beach sands in widely separated areas has been derived from parent lithologies with similar characteristics. If the end members were to be interpreted as geographically distinct fluvial source assemblages, these subsidiary modes would have to be regarded as noise or artefacts of the modelling procedure. However, a more flexible interpretation can be made by taking into account that the end members can only be modelled under the assumption of linear independence. End members whose compositions can be expressed as mixtures of other end members cannot be reconstructed from the data. Following this view, the subsidiary modes of end members 1 and 4 can be interpreted as contributions of minor sediment sources, whose compositions can be expressed as non-negative mixtures of the modelled end members. Increasing the number of end members does not necessarily resolve the contributions of such sources, unless their linear dependence is an artefact induced by the reduction of the dimensionality of the input data. True linear dependence (collinearity) in compositional space is a fundamental problem that cannot be resolved by inverse methods (see also Harvey & Lovell, 1992).

If the geographical pattern of end-member contributions to the beach sands is interpreted in terms of sedimentary petrological provinces, the end-member compositions must represent sediments derived from four major drainage basins or source areas. The lithological characteristics of these hypothetical source areas can be inferred from the end-member compositions depicted in Figure 8 and Table 3. End member 1 consists predominantly of carbonate (micritic, sparitic, and dolomitic grains), and contains minor amounts of quartzose grains. Such sediments can only be derived from a source area that consists almost exclusively of carbonate rocks. End member 2 contains a large amount of quartz, feldspar, and volcanic lithics. Small amounts of carbonate lithics point to the presence of sedimentary rocks in the source area. An unknown, but presumably small proportion of the other grain types could have been reworked from these sedimentary rocks, but igneous, metamorphic and volcanic rocks are expected to be the dominant lithologies in the source area. End member 3 appears to be derived from a mixed source area in which sedimentary, igneous and metamorphic rocks are present. The source area of end member 4 consists of various sedimentary parent lithologies; carbonates and siliciclastic rocks are expected to be the dominant lithologies in this area, whereas volcanites and metamorphites appear to be absent.

8. Validation of modelling results

The mixing model presented above would represent the final outcome of this study, if the sediments had been derived from an ancient basin fill for which no additional information was available. In practice, even a limited amount of additional information can provide a considerable refinement of mixing-model interpretations. For instance, the spatial pattern of end-member contributions can be compared to isopach maps, palaeocurrent measurements, and sedimentary facies associations, in order to judge whether the palaeogeographic locations of sediment sources predicted by the model conform to other lines of evidence. If the composition of the sand-sized portion of a basin fill accurately reflects its parent lithologies, the interpretation of end-member compositions can be strengthened by comparing the end members to synthetic mixtures of rocks from the hinterland, which may be present in the basin fill in the form of conglomerate clasts. If the data set consists of point-counting results, as in this case, compositions of possible source rocks could be quantified by point counting of crushed conglomerate clasts. Ideally, synthetic mixtures of parent lithologies closely approximate the end-member compositions, allowing one to estimate the contributions of each parent lithology to the size fraction studied. For the Po-delta plain and adjacent coastal zones, a wealth of additional information is available for testing the model predictions. The results of three tests will be presented; each of which requires a higher level of a priori knowledge.

9. The first test: matching of possible source compositions

The modelling results can be examined in more detail by comparing the end members to the sediments supplied by the rivers debouching in this area. Gazzi et al. (1973) analysed the compositions of possible source sediments, in order to unravel the dispersal pattern along the Adriatic shoreline. This second data set consists of 15 fluvial samples and 1 sample from the Miocene Marnoso-Arenacea Formation exposed along the Adriatic coast. A (16×10) array of possible source compositions was obtained by the same pre-processing steps used for the beach sands (Table 1). The sampling stations of these possible source compositions are depicted in Figure 4 (samples with suffix ‘F’ or ‘P’). Unfortunately, only one sample is available from each possible source. It has been observed that fluvial sediments in this area display a considerable compositional variation (Jobstraibizer & Malesani, 1973; Gazzi et al., 1973), indicating that the source compositions are also subject to errors of sampling, measurement, and natural heterogeneity. The possible source compositions were expressed as non-negative mixtures of the end members calculated from the beach sands. In order to account for errors in all of the compositions used for this test, the end-member compositions were appended to the data set of source compositions. Subsequently, a second mixing model was constructed in which all source compositions were expressed as non-negative mixtures of these four end members. The mixing proportions associated with each source composition express the similarity of each source assemblage to the modelled end members. Sources that display the strongest similarity to the modelled end members can be regarded as the most likely suppliers of sediment to beaches in which the modelled contributions of these end members is largest.

A plot of the calculated similarities against the approximate river-mouth locations (Fig. 9) shows that end member 1 is most similar to sediments of the Isonzo, Tagliamento, Livenza, and Piave rivers, which debouch in the north-eastern part of the study area (province A). End member 2 closely resembles sediments of the Brenta and Adige rivers. Coastal province B coincides with the locations of the river mouths. End member 3 displays a strong affinity to sediments supplied by the Po, Reno, Lamone, and Savio rivers. The geographical extent of province C also corresponds to the locations of the river mouths. End member 4 closely resembles sediments of the Marecchia, Conca and Foglia rivers, which contributed to the beaches of province D. A number of sediment sources in the transitional area between provinces C and D are more difficult to classify in terms of compositional similarity to a particular end member. These are the Uniti and Rubicone rivers, and the sandstone cliffs of Punta Gabicce, in which the Marnoso-Arenacea Formation is exposed. Sediments supplied by these sources can be interpreted as mixtures of end members 3 and 4, with smaller amounts of end members 1 and/or 2. Contributions of sediments from these sources could at least partly explain the complicated pattern of mixing observed in the transition zone of provinces C and D.

The overall spatial coherence of probable sediment sources and their corresponding sedimentary petrological provinces convincingly demonstrates the general validity of the model predictions. The beach sands closely resemble the compositions of their fluvial sources, indicating that compositional modifications due to physical and chemical processes (i.e. selective transport, mechanical and chemical weathering) are of minor importance. Furthermore, alongshore dispersal of beach sands appears to be of limited extent, in view of the lack of offset between locations of river mouths and their corresponding sedimentary petrological provinces. The only clear evidence of net alongshore dispersal is provided by the sediments of province A, which extend far southwestward of their nearest likely source. The drainage basins that provided the beach sands of the four sedimentary-petrological provinces are displayed in Figure 10.

10. **The second test: matching of predicted fluvial input**

A compilation of physiographic data on the fluvial drainage systems is presented in Table 4. The Po River is by far the largest river in terms of drainage basin area, discharge, and sediment load. Second largest (but an order of magnitude smaller than the Po) is the Adige River. Together, these two rivers account for one third of the total freshwater input into the Adriatic Sea (Pettine et al., 1985). The other rivers are far less important in terms of liquid discharge and sediment load. They supply comparable amounts of sediment to the Adriatic beaches, except for the Reno river (Milliman & Syvitski, 1992), which carries an unusually large solid load relative to the other small rivers in this area. The physiographic data in Table 4 have been combined with the results of the first matching exercise to predict a weighted average composition of sediment supplied by each source area to its corresponding province. This test is much more straightforward than the first, because the source assemblages are not forced to conform to a predefined set of compositions, such as the modelled end members. Instead, fluvial source assemblages and physiographic data are combined to arrive at an independent prediction of sediment composition. Proportional contributions of individual rivers of each province were estimated from drainage area, discharge, and solid load. These estimates, which reflect the present conditions of rivers debouching in this area, were obtained by averaging values reported by various authors. Three source assemblages were excluded from this test because of their lack of affinity to a particular end member (Uniti and Rubicone rivers, and the sandstone cliffs of Punta Gabicce). The estimated weights (mixing proportions) and the compositions of the synthetic river mixtures are depicted in Table 5 and Figure 11, respectively.

Figure 11 shows that end member 1 is reasonably well approximated by a synthetic mixture of fluvial sands from province A, although the former contains more sparite and less chert than the latter. End members 3 and 4 are very well approximated by synthetic mixtures of sands from provinces C and D, respectively. In view of possible errors in the source compositions, it is concluded that end members 1, 3, and 4 accurately reflect the compositions of sands supplied by the Dolomites, the Po river basin, and the Northern Apennines, respectively. End member 2 is poorly approximated by a synthetic mixture of Adige and Brenta sands.

The lack of fit of end member 2 to the synthetic Adige-Brenta mixture points to a discrepancy between model predictions and the actual situation. Figure 6 shows that the predicted volcanic-lithics content of beach sands between stations 21 and 25 is systematically lower than the observed content in province B.

This strongly suggests that beach-sand compositions in province B are not well approximated by the mixing model. Furthermore, the angular deviations of beach sands in province B are systematically larger in the final model than in the original four-dimensional approximation of the data. A synthetic Adige-Brenta mixture to which Po sand has been added shows a much stronger resemblance to end member 2, indicating that end member 2 can be loosely interpreted as a mixture of sands supplied by the Brenta-Adige rivers and the Po river basin. Hence, the modelled composition of end member 2 is not sufficiently extreme for an adequate approximation of the volcanic-lithics content in beach sands of province B, implying that compositional differences between Po-type sediments (similar to end member 3) and Adige-Brenta-type sediments (similar to end member 2) have been underestimated by the model.

The lack of fit of end member 2 may be attributed to the fact that Adige-Brenta input is well represented in only 5 out of 53 observations. The iterative algorithm, which produces conservative end-member estimates based on the collective properties of the data set, did not increase the size of the mixing space sufficiently to construct an end-member composition that encloses all of the observations from province B. This implies that the true contributions of Adige-Brenta sands to the coastal sands in the other provinces are lower than the modelled contributions of end member 2. The lack of fit of end member 2 illustrates a possible disadvantage of the robust estimation procedure employed by the end-member-modelling algorithm. However, the conservative estimation procedure guarantees the geological reasonableness of hypothetical end-member compositions, as shown above. In the absence of a priori knowledge of the area under study, conservative estimates are preferred over extreme end-member estimates. Extreme end-member estimates improve the overall fit of the model to the data, but may not be interpretable in geological terms.

11. **Late Holocene evolution and present situation**

Spatial mixing patterns of sands along wave-dominated coasts are the result of multiple cycles of erosion, transport, and deposition. The modelled mixing patterns thus represents the cumulative effects of long-term sediment transport. In comparison to the time scale of these processes, present physiographic characteristics of drainage basins, locations of river mouths, fluvial sediment compositions, and recent littoral drift patterns can be regarded as snap shots. Discrepancies between long-term dispersal patterns and snap shots of the present situation in the study area could be due to recent anthropogenically induced environmental changes in this densely populated region. The Holocene evolution of the Po basin has been reconstructed in detail (Pigorini, 1968; Nelson, 1970; Van Straaten, 1970; Rizzini, 1974; Colantoni et al., 1979; Gandolfi et al., 1982; Dal Cin, 1983), enabling a comparison of modelling results with the past 3000 years of coastal evolution. The late Holocene evolution of the Po basin is characterised by progradation of the shoreline, which started around 3000 yr BP and has continued up to recent times (Fig. 12). The presently widespread erosion of Northern Adriatic beaches is ascribed to a profound anthropogenically induced decrease of fluvial bedload (CNR, 1976; Bondesan et al., 1978; Brambati et al., 1978; Dal Cin, 1983). Widespread coastal retreat confirms the idea that model predictions could partly reflect ancient compositional signals, due to reworking of older deposits. Sedimentological research and historical records of coastal-plain evolution shed more light on the origin of deposits currently subject to reworking along the Northern Adriatic coast.

From the Roman age to 1200 AD, the most important branches of the Po were located south of the present delta. The Po of Ferrara consisted of two main distributaries: the Po di Volano and the Po di Primaro (the present Reno river), which built cuspate deltas (Fig. 12; Nelson, 1970; Gandolfi et al., 1982). In the same period, the mouth of the Adige river shifted southward to its present position. Petrographic studies have shown that southward dispersal of Adige and Po sands was much more extensive than it is at present (Rizzini, 1974; Gandolfi et al., 1982). Around 1200 AD, a natural diversion occurred at Ficarolo, which resulted in an abrupt northward shift of the Po mainstem. Discharge of the southerly Po di Ferrara branches decreased, and a new cuspate delta developed just north of the present Po delta, close to the mouth of the Adige river (Nelson, 1970; Gandolfi et al., 1982). This delta was built by two distributaries: the Po di Tramontana and Po di Levante (Fig. 12). It also received sediment from minor distributaries of the Adige River (Dal Cin, 1983). Around 1600 AD, the rapid growth of this delta threatened to block the ports of the Venetian Republic with silt, and a diversionary canal was dug at Porto Viro to correct this situation. From this time onward, the evolution of the Po delta has been largely controlled by human activity. In the second half of the 18th century, the largely inactive Po di Primaro branch was connected with the drainage basin of the Reno River (Bondesan et al., 1978). The rapid advancement of the modern lobate Po delta in the past 400 years is attributable to deforestation of the drainage basin, the construction of artificial levees, and the confinement of its deposits to a small area (Nelson, 1970; Dal Cin, 1983).

The present situation of the Northern Adriatic coastal zone can be summarised as follows. In the northern part of the area, erosion balances accretion, and alongshore dispersal of sands is mainly to the west and southwest (Brambati, 1970; Brambati et al., 1977, 1978; Gandolfi et al., 1978a). The beaches near the Brenta and Adige rivers, just north of the Po delta, are presently subject to erosion (CNR, 1976). Beach sands in the area between the Po delta and the mouth of the Adige River are derived from reworked ancient sediments. They do not represent recent input of the Adige-River sediments (Dal Cin, 1983). The pronounced seaward extension of the modern lobate delta effectively precludes southward dispersal of sediments from province B (Gandolfi et al., 1982). The Po delta, which has prograded rapidly in the past four hundred years due to human intervention, is currently retreating in many areas, and regaining its original cuspate morphology. Sands are dispersed from the main river mouth towards the northwest and southwest by divergent alongshore transport (Dal Cin, 1983). The area south of the Po delta is characterized by northward littoral drift. Erosion prevails over accretion (Gazzi et al., 1973; Rizzini, 1974; Dal Cin, 1983). Detailed studies in this area indicate that coastal erosion is especially prominent in the area around the mouths of the Reno and Savio rivers. Beach accretion occurs in the area just south of the present Po delta, due to the northward dispersal of reworked sands of the Reno mouth (Bondesan et al., 1978). The present regional dispersal patterns converge in the areas directly north and south of the Po delta (Fig. 12; Gazzi et al., 1973).

12. The third test: historical evolution and present conditions

How do the modelling results fit the historical evolution of the Po delta and modern sediment dispersal patterns? This is most conveniently discussed for each end member (and its corresponding province) separately.

End member 1 can be reasonably well approximated by a synthetic mixture of fluvial sediments supplied by the hinterland of province A. This province extends beyond the lagoon of Venice up to the Brenta river mouth, indicating that west to south-westward dispersal prevails in this area. The predicted alongshore dispersal is confirmed by sedimentological observations (Brambati, 1970; Brambati et al., 1977, 1978; Catani et al., 1978; Gandolfi et al., 1978a). Local contributions of end member 4 to beaches in province A reflect the compositional similarity of certain fluvial sources (Tagliamento and Piave rivers) to sediments derived from the Northern Apennines, which cannot be resolved on this scale. In conclusion, there are no discrepancies between model predictions and other lines of argument.

End member 2 cannot be satisfactorily approximated by a synthetic mixture of Brenta and Adige sands, as shown in the second validation test. Addition of Po sands improves the goodness-of-fit considerably, indicating that the modelled composition of end member 2 is not extreme enough to provide an accurate approximation of Adige-Brenta input. Consequently, the contribution of end member 2 to beach sands of provinces C and D, which suggests a southward dispersal of Adige-Brenta sands, must be regarded as a modelling artefact. Sedimentological data indicate a northward littoral drift in the northern part of province B, whereas bi-directional transport characterises the southern part of this province (CNR, 1976; Dal Cin, 1983). Beach sands in the southern part of province B are inferred to have been derived from reworking of the ancient Po di Tramontana delta (Dal Cin, 1983). Because this delta was fed by the Po and Adige rivers, the composition of beach sands in the southern part of province B cannot be considered as conclusive evidence for south-eastward dispersal of Adige sands, as suggested by Gazzi et al. (1973) and Gandolfi et al. (1978b).

End member 3 can be well approximated by a synthetic mixture of Po sands and minor amounts of Reno, Lamone, and Savio sands. There is little doubt that the sands of the present Po-delta, which forms the northern part of province C, have been supplied by the Po. In fact, it can be assumed that virtually all of the sand carried by the Po branches in the past 400 years has contributed to the construction of the modern lobate delta (Gazzi et al., 1973; Dal Cin, 1983). A northward littoral drift is present in the area south of the Po delta (Rizzini, 1974; Bondesan et al., 1978; Dal Cin, 1983), implying that the Po river is an unlikely source of beach sands in this area. The origin of Po-type sands in the southern part of province C must be largely attributed to reworking of ancient Po di Primaro deposits. The ancient Po di Primaro delta corresponds to the modern Reno river mouth, which is presently subject to erosion (Bondesan et al., 1978). Coastal progradation in the area of the former Po di Volano delta, further to the north (Fig. 12), reflects the northward littoral drift of these reworked Po deposits (Bondesan et al., 1978). It seems likely that the beaches in the southern part of province C have also been fed by sands of the Lamone and Savio rivers, that are compositionally similar to end member 3. A contribution of reworked Savio sands to the beaches of province C is strongly suggested by sedimentological and historical data, which indicate erosion in front of the river mouth (Bondesan et al., 1978). In conclusion, there is no evidence for a southward dispersal of sediments in province C, as suggested by the modelling results. The alongshore variations of end-member contributions in the southern part of province C merely reflects the reworking of ancient Po-type sands.

End member 4 closely fits the synthetic mixture of Marecchia, Conca, and Foglia sands, indicating that beach sands of province D have been supplied by recycling of sedimentary rocks of the Northern Apennines. The beach sands near Rimini (stations 47 to 49) show a distinct affinity to end member 1 (Fig. 7). This affinity has been attributed to a local supply of sands from the Marecchia river, which contain a large proportion of carbonate (Gazzi et al., 1973). Detailed heavy-mineral studies of Holocene beaches located between stations 47 and 51 (Rizzini, 1974) strongly suggest that a considerable proportion of sands in this area is derived from reworking of relict shelf sands, which were transported towards the shoreline during the Holocene sea-level rise. These relict shelf sands were supplied by the Po River during the previous Pleistocene lowstand, when the entire Northern Adriatic shelf had become an extensive alluvial plain (Pigorini, 1968; Van Straaten, 1970; Colantoni et al., 1979). The Po signature does not clearly show up in the modelling results, although contributions of end member 3 to the beaches between stations 50 and 51 indeed show a small peak (Fig. 7). Northward alongshore dispersal is of limited extent in the southern part of province D. Model predictions closely match the actual situation.

13. Comparison with plate-tectonic provenance models

In order to compare the results of the provenance modelling experiment with the plate-tectonic provenance models developed by Dickinson and co-workers (summarised by Dickinson, 1985, 1988), the four end-member compositions and the mean composition of the Adriatic beach sands have been plotted in the QtFL and QmFLt diagrams of Dickinson (1985). In the terminology of Ingersoll (1990), the end-member compositions and the averaged composition would have to be regarded as second and third-order compositions, respectively. The grain types used in the modelling experiment were amalgamated to provide a reduced set of variables in agreement with the Dickinson (1985) conventions (Table 6). The proportions of monocrystalline and polycrystalline quartz were not recorded separately by Gazzi et al.

(1973). For lack of a better alternative, all quartz has been classified as monocrystalline. The Adriatic beach sands contain an appreciable proportion of extrabasinal carbonate grains (detrital lime clasts), which are commonly ignored because of their susceptibility to weathering and diagenesis (Dickinson, 1985). However, Mack (1984) and Zuffa (1980, 1987) have shown that such extrabasinal grains may help to clarify provenance relationships in certain cases. The third-order provenance fields of Dickinson (1985) are shown in Figure 13. The compositions obtained by excluding the extrabasinal carbonate grains from the modelling results are depicted on the left-hand side of Figure 13. The total lithic population has been employed for the construction of provenance diagrams on the right-hand side of this figure. The actual plate-tectonic provenance types (sensu Dickinson, 1985) and source-area characteristics of the four end-member assemblages have been summarised in Table 7. This table also shows the extent to which the ternary provenance diagrams of Dickinson (1985) are capable of recognising the plate-tectonic setting of the four end-member assemblages.

The QtFL diagram correctly identifies the recycled-orogen provenance of end members 1, 2, and 3. End member 4 contains too few non-quartzose lithics (L) to be classified correctly: the sedimentary parentage of Northern Apenninic input is not recognised (although the location of end member 4 is very close to the recycled-orogen field). Adding the carbonate lithics (Lc) to non-quartzose lithics (L) produces a catastrophic misinterpretation of end members 1 and 4, and of the third-order composition. The QtFL diagram is unable to cope with extrabasinal lime clasts, because all lithic sands containing small proportions of quartz and feldspar are classified as magmatic-arc sediments. In this case, the use of QmFLt diagrams requires the assumption that all quartz is monocrystalline; the locations of data points in the QmFLt diagram are therefore not very reliable. Apart from end member 2 and the mean composition of the data set, none of the other compositions is correctly classified in this diagram. In view of the high abundance of carbonate lithics, possible errors resulting from the assumption that all quartz is monocrystalline are unlikely to affect the location of data points in the QmFLt+Lc diagram. The second-order parentage of all end members is correctly identified by the QmFLt+Lc diagram. The mean composition of the beach sands is also satisfactorily classified.

Predictive regions of compositional variation have been constructed by connecting the end members plotted in the ternary diagrams by a set of lines (Fig. 13). This predictive region represents the projection of the three-dimensional mixing space (a tetrahedron in compositional space) onto the constant-sum plane of the ternary diagram. The average composition of any sand suite from one of the second-order drainage basins or the beaches in the study area should plot in this predictive region, if the lower-order parentage has been accurately reconstructed. This approach potentially eliminates the scale problem in current provenance models and simultaneously improves the resolution of the model, by enabling a comparison of second-order and third-order provenance. Furthermore, the predictive region constructed from the inverse modelling results is superior to the polygonal field of dispersion or variation commonly employed in sedimentary provenance studies, whose physical and statistical meaning is obscure (Philip et al., 1987).

14. **Discussion and conclusions**

The comparison of model predictions and the actual situation indicates that the performance of the end-member-modelling algorithm is quite satisfactory. Three out of four modelled end members closely approximate the compositions of sediments shed by the major source areas of the beaches studied. A fourth end member displays a clear affinity to its actual source composition. The compositional signature of this end member appears to have been underestimated by the robust estimation procedure, because of its small overall contribution to the data set. In spite of the lack of fit, its composition is geologically reasonable. On the scale of this regional survey, small collinear sediment sources can be detected (e.g., the Marecchia River in province D), but their contributions cannot be resolved. The modelled alongshore variation of beach sands, expressed as proportional end-member contributions, is in general agreement with present dispersal patterns and historical records of the area. A detailed analysis of historical records and present littoral drift shows that the overall southwest to southward dispersal suggested by the modelling results must be attributed to shifting of Po distributaries in historical times. Recycling of these ancient Po sediments is related to the present decrease of fluvial sediment supply, and the widespread coastal retreat in the area studied.

The results of the validation tests demonstrate that compositional variation of the Northern Adriatic beach sands can be adequately represented by a linear mixing model, in spite of the fact that sands along this wave-dominated coast consist predominantly of chemically and mechanically unstable grains, that are highly susceptible to selective destruction. Mechanical destruction and/or chemical weathering of coastal sands have not noticeably affected the modelling results. A reason for the apparent lack of compositional modification is the high rate of sediment supply that prevailed during the late Holocene in this area. The generally high rate of Holocene coastal progradation indicates that the average sand grain spends only a brief interval of time in transport before it is deposited and buried. However, these conditions have changed drastically in the recent past, as a result of human activity.

The present beach-sand compositions provide a time-averaged signal of late Holocene sediment dispersal, because ancient sands are being eroded and re-introduced into the dispersal system. The temporal resolution appears to be of the order of hundreds to thousands of years, reflecting the balance between overall coastal progradation and retrogradation. This balance, in turn, is determined by the interplay of local sediment supply and base-level variations. Dispersal patterns can only be studied because the temporal resolution of provenance signals is (at least) an order of magnitude lower. Highest recorded frequencies of provenance signals in ancient sands are of the order of 20 to 25 kyr (Weltje & De Boer, 1993). On average, compositional signatures of fluvial sediments supplied by first and second-order drainage basins are inferred to be stable over much longer intervals (Velbel & Saad, 1991).

Many (if not all) sedimentary basin fills are mixtures of sediments supplied by different sources. This applies to spatial variation within a given time-slice, but it can also apply to temporal compositional variation of sediments supplied by a single source area. A method for apportioning mixtures to their respective sources is a prerequisite for the unravelling of complex patterns of provenance and dispersal. A simple graphical representation of the mixing model in the plate-tectonic provenance diagrams clearly demonstrates the usefulness of this approach to the analysis of compositional variation. The general solution to the mixing problem in sedimentary provenance studies provides a major step forward towards successful mass-balancing and three-dimensional forward modelling of sedimentary basin fills (e.g., Leeder, 1991). Although the inversion algorithm is a powerful tool for unravelling complex patterns of compositional variation, it cannot provide an explanation of the observed patterns of compositional heterogeneity. The mixing model merely represents the output of a complicated data manipulation procedure. It can only be judged by the modeller if the estimated mixing parameters are also geologically feasible.

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Table 1. Pre-processing of input data (from Gazzi et al., 1973) for the end-member modelling experiment.

Original variables	New variables	Codes
Quartz	Quartz	Q
K-feldspar Albite Plagioclase	Feldspar	F
Acidic volcanites Basic volcanites	Volcanic lithics	Lv
Serpentine Serpentine schist Phyllite	Metamorphic lithics	Lm
Shale Calcareous siltstone Non-calcareous siltstone	Siliciclastic lithics	Ls
Sparite	Sparite	Sp
Micrite	Micrite	Mi
Bioclasts	Bioclasts	Bi
Dolostone	Dolostone	Do
Chert	Chert	Ch
Micas and other	(excluded)	-

Table 2. Goodness-of-fit statistics for linear model with four reference vectors and final mixing model. The fit of the model to the input data decreases slightly after imposing non-negativity constraints on mixing proportions.

Coefficients of determination	Linear model	Mixing model
Quartz	0.97	0.97
Feldspar	0.94	0.94
Volcanic lithics	0.72	0.66
Metamorphic lithics	0.64	0.59
Siliciclastic lithics	0.55	0.48
Sparite	0.84	0.84
Micrite	0.96	0.95
Bioclasts	0.74	0.66
Dolostone	0.71	0.68
Chert	0.44	0.37
Mean coefficient of determination	0.75	0.71
Mean angular deviation (degrees)	8.45	8.95
Normality test of residual logratio distribution:		
Number of residual logratios		36
Chi-square value of distribution (df = 2)		4.28
Associated probability		0.88

Table 3. Modelled end members of Northern Adriatic beach sands (see also Fig. 8).

Grain types	End-member compositions (%)			
	1	2	3	4
Quartz	4.3	52.3	49.1	23.4
Feldspar	0.0	25.6	26.1	9.7
Volcanic lithics	0.0	11.3	0.4	0.0
Metamorphic lithics	1.3	2.3	4.1	0.0
Siliciclastic lithics	1.1	0.0	2.1	1.0
Sparite	38.2	2.6	11.8	25.0
Micrite	41.8	0.0	3.1	22.3
Bioclasts	0.0	4.5	3.3	14.4
Dolostone	10.7	0.0	0.0	0.2
Chert	2.5	1.4	0.0	4.0

Table 4. Physiographic characteristics of fluvial systems, based on various sources (Nelson, 1970; CNR, 1976; Hendershott & Rizzoli, 1976; Brambati et al., 1977; Bondesan et al., 1978; Catani et al., 1978; Dal Cin, 1983; Bortoluzzi et al., 1984; Pettine et al., 1985; Milliman & Syvitski, 1992). Values based on single reference only are marked with (*).

Fluvial systems	Code	Basin area (10^3 km^2)	Mean discharge (m^3 s^{-1})	Solid load (10^6t yr^{-1})
Isonzo Tagliamento Livenza Piave Brenta	Is Ta Li Pi Br	1.0 - 2.0	80 - 100	< 1
Adige	Ad	12.2	220	1
Po	Po	63.6	1500	14
Reno	Re	3.8	80 (*)	2 (*)
Lamone Uniti Savio Rubicone Marecchia Conca Foglia	La Un Sa Ru Ma Co Fo	0.5 - 1.5	50 – 80	< 1

Table 5. Estimated synthetic mixtures of fluvial sands delivered to the Northern Adriatic coast. Proportional contributions based on physiographic data of Table 4.

Fluvial systems	Synthetic mixtures of fluvial sands				
	1	2	2+Po	3	4
Isonzo	0.25				
Tagliamento	0.25				
Livenza	0.25				
Piave	0.25				
Brenta		0.25	0.25		
Adige		0.75	0.50		
Po			0.25	0.65	
Reno				0.15	
Lamone				0.10	
Savio				0.10	
Marecchia					0.33
Conca					0.33
Foglia					0.33

Table 6. Summary of actual source-area characteristics and empirical classification of provenance types according to the ternary provenance diagrams of Dickinson (1985). Empirical classification of end members and sample mean: + = correct; - = incorrect; +/- = nearly correct.

End member Province	1 A	2 B	3 C	4 D	Sample mean
Orogenic source area	S. Alps	S. Alps E. Alps	E. Alps W. Alps N. Apennines	N. Apennines	Alps Apennines
Principal parent lithologies	Sedimentary (carbonate)	Sedimentary Igneous Metamorphic	Sedimentary Igneous Metamorphic	Sedimentary (siliciclastic)	Sedimentary Igneous Metamorphic
Provenance types (cf. Dickinson, 1985)	Recycled (lithic)	Recycled or mixed	Recycled or mixed	Recycled (transitional)	Recycled or mixed
QtFL	+	+	+/-	-	+
QtFL+Lc	-	+	+	-	-
QmFLt	+/-	+	-	+/-	+/-
QmFLt+Lc	+	+	+	+	+

Table 7. Simplification of modelled end-member compositions for display in ternary provenance diagrams of Dickinson (1985).

New variables	Definitions	Codes
Monocrystalline (?) quartz	Q	Qm
Total quartzose grains	Q + Ch	Qt
Feldspar	F	F
Non-quartzose lithics	Lv + Lm + Ls	L
Non-carbonate lithics	Lv + Lm + Ls + Ch	Lt
Extrabasinal lime clasts	Sp + Mi + Bi + Do	Lc

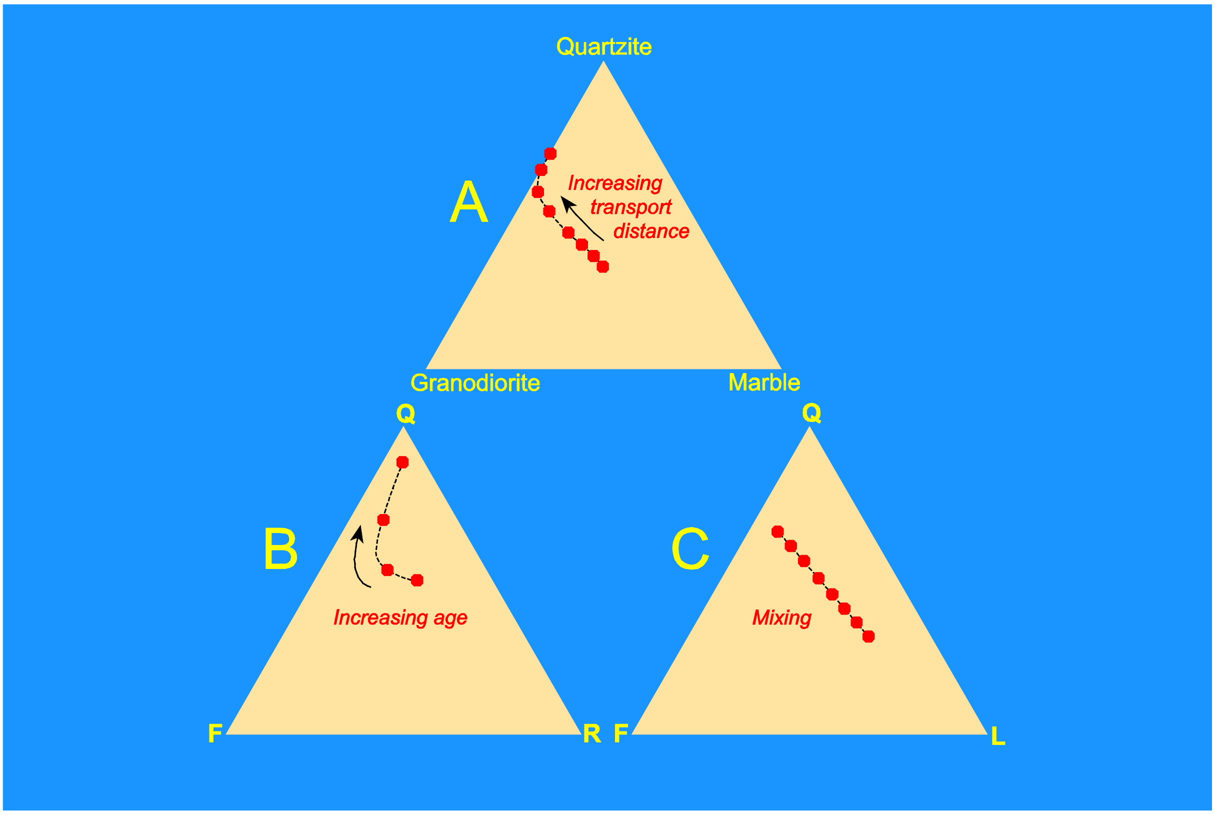

Figure 1: Examples of compositional trends. A) Non-linear compositional modifications caused by abrasion of polylithologic conglomerate-clast assemblage (data recalculated from Abbott & Peterson, 1978). B) Non-linear compositional modifications caused by in situ weathering of sands in a point bar of the Amazon Basin (data from Johnsson & Meade, 1990). C) Linear compositional trend caused by mixing of two source compositions.

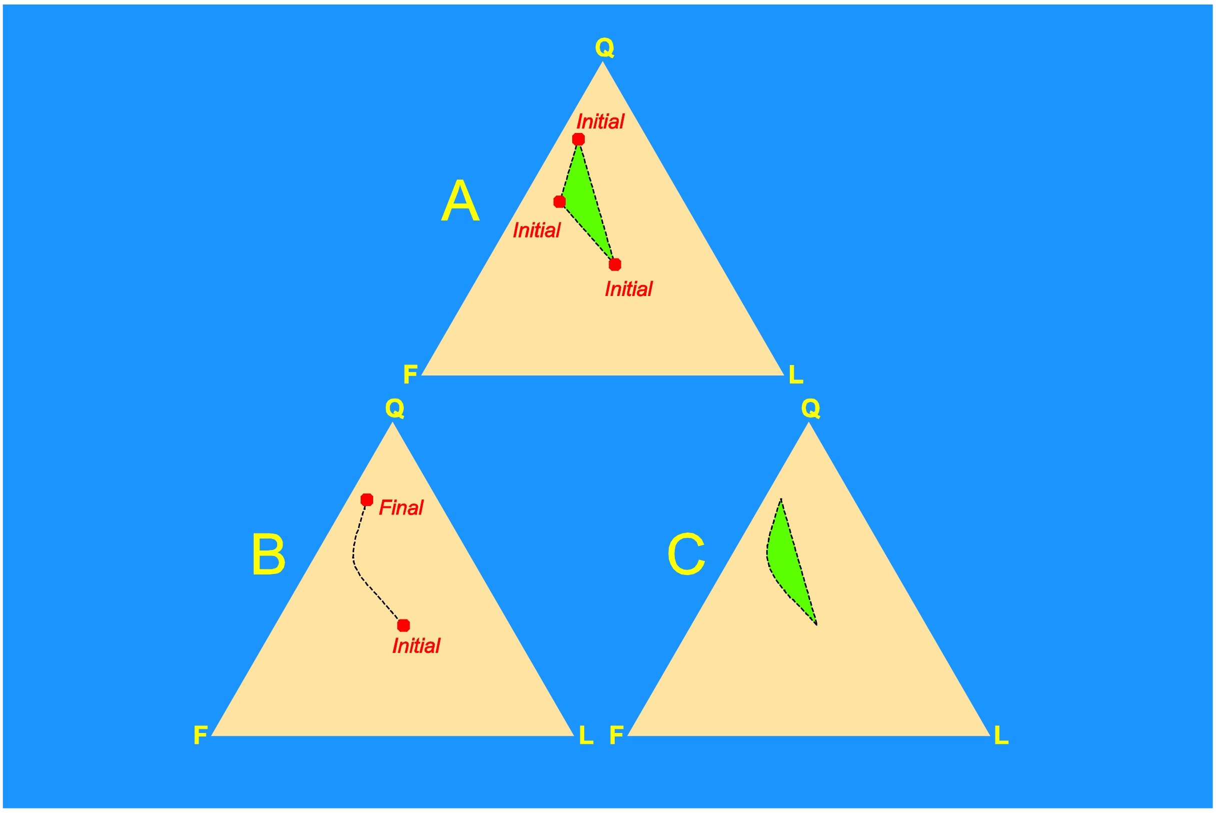

Figure 2: Interpretation of complex patterns of compositional variation. A) Random ternary mixing produces a triangular area of compositions, the vertices of which are formed by the source compositions. B) Vertical compositional variation in a soil profile, characterised by a curved trend. C) Physical mixing of the soil sediments of Figure 2B obscures the non-linear trend in the data and produces a ternary mixing pattern indistinguishable from Figure 2A.

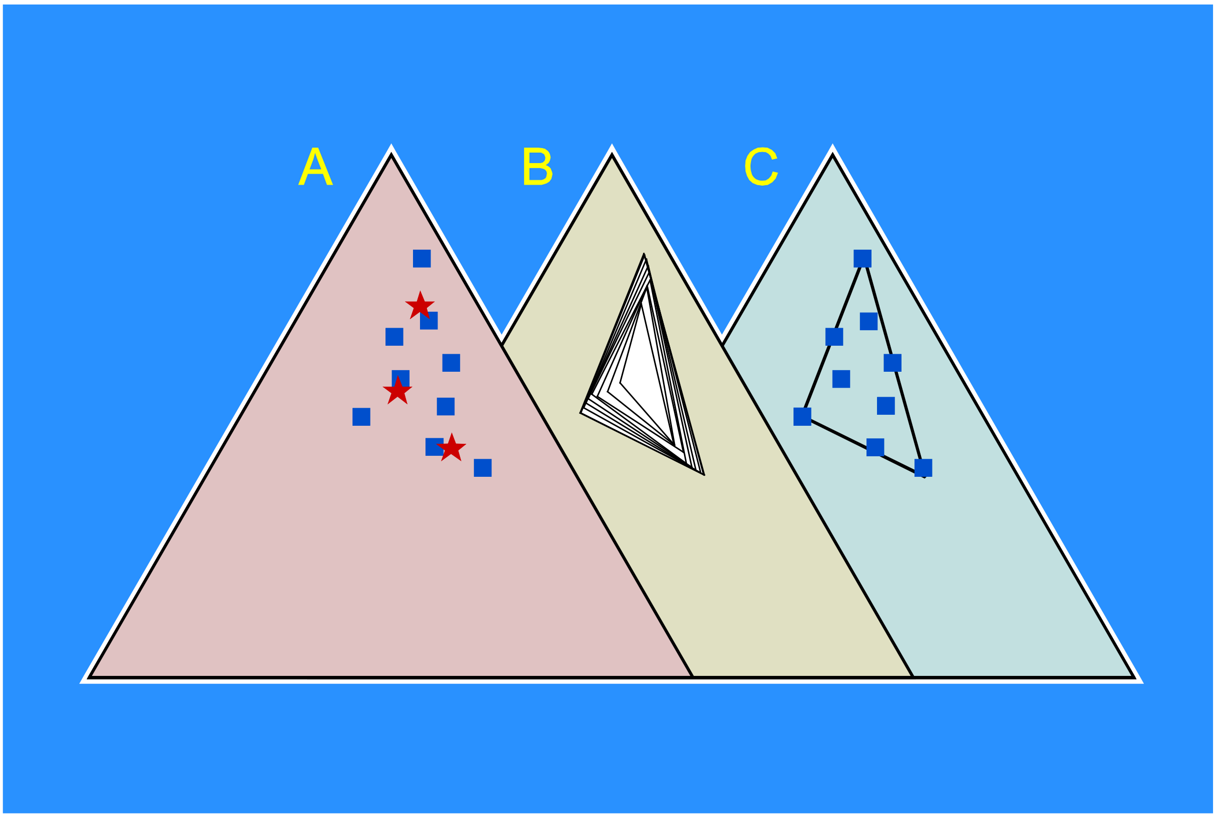

Figure 3: Example of iterative end-member modelling for a simple case of ternary mixing (Weltje, 1994). A) Data set generated by randomly mixing three source compositions. The initial end-member estimates used by the iterative algorithm (the three centres of mass of the data set), are marked by asterisks. B) The mixing models calculated in subsequent iteration cycles are represented by a series of triangles; each new triangle encloses the former. C) The final mixing model accurately identifies the structure in the data, i.e. the source compositions are almost perfectly reproduced.

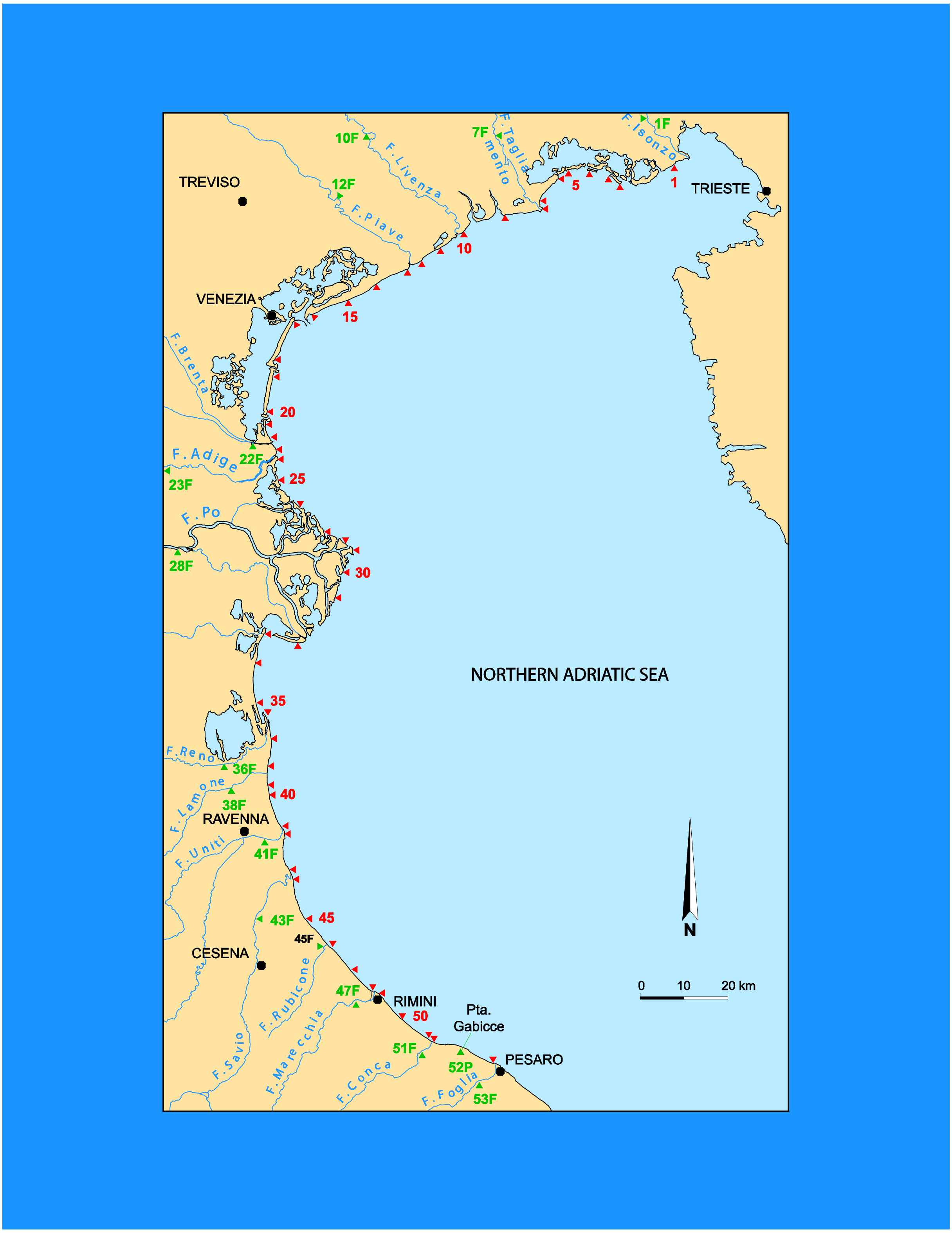

Figure 4: Simplified map of the Northern Adriatic sea, showing coastal and fluvial sampling stations (after Gazzi et al., 1973).

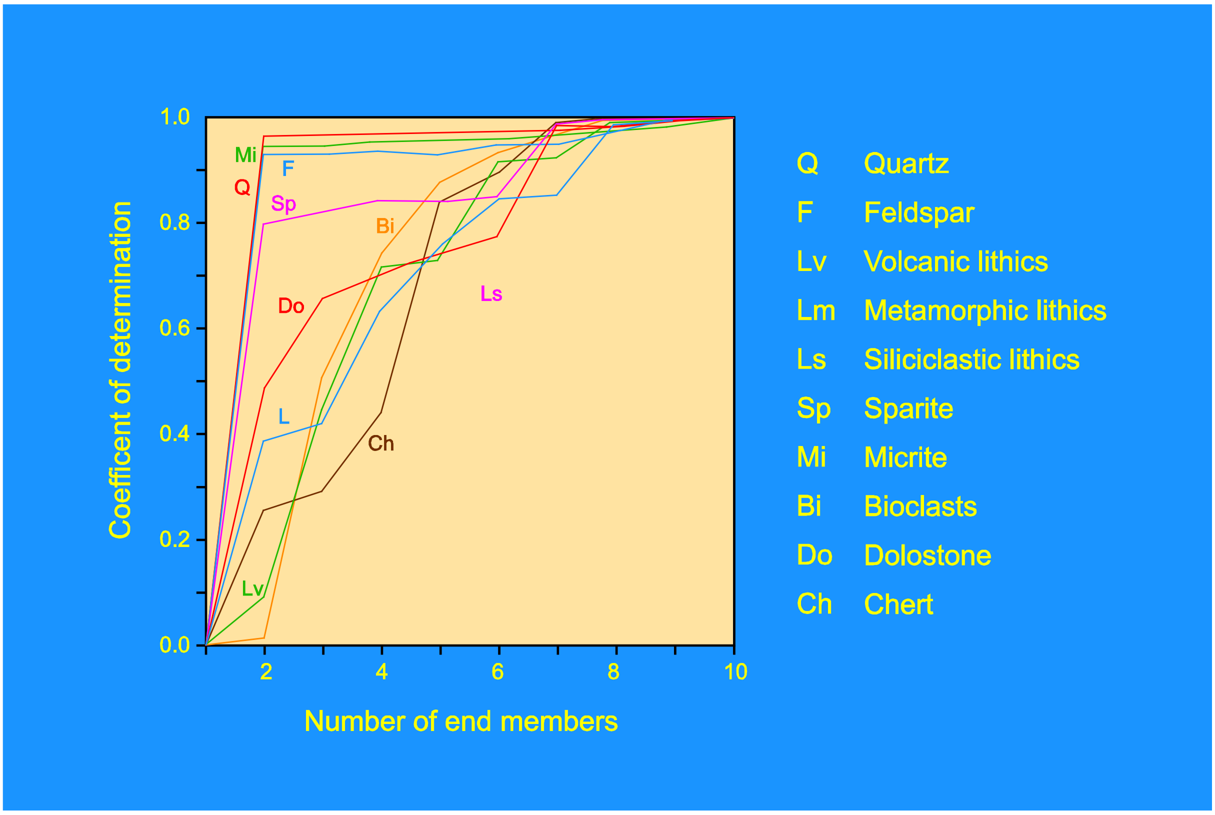

Figure 5. A) Coefficients of determination for each variable. Four end members are the minimum number required for an adequate description of compositional variation.

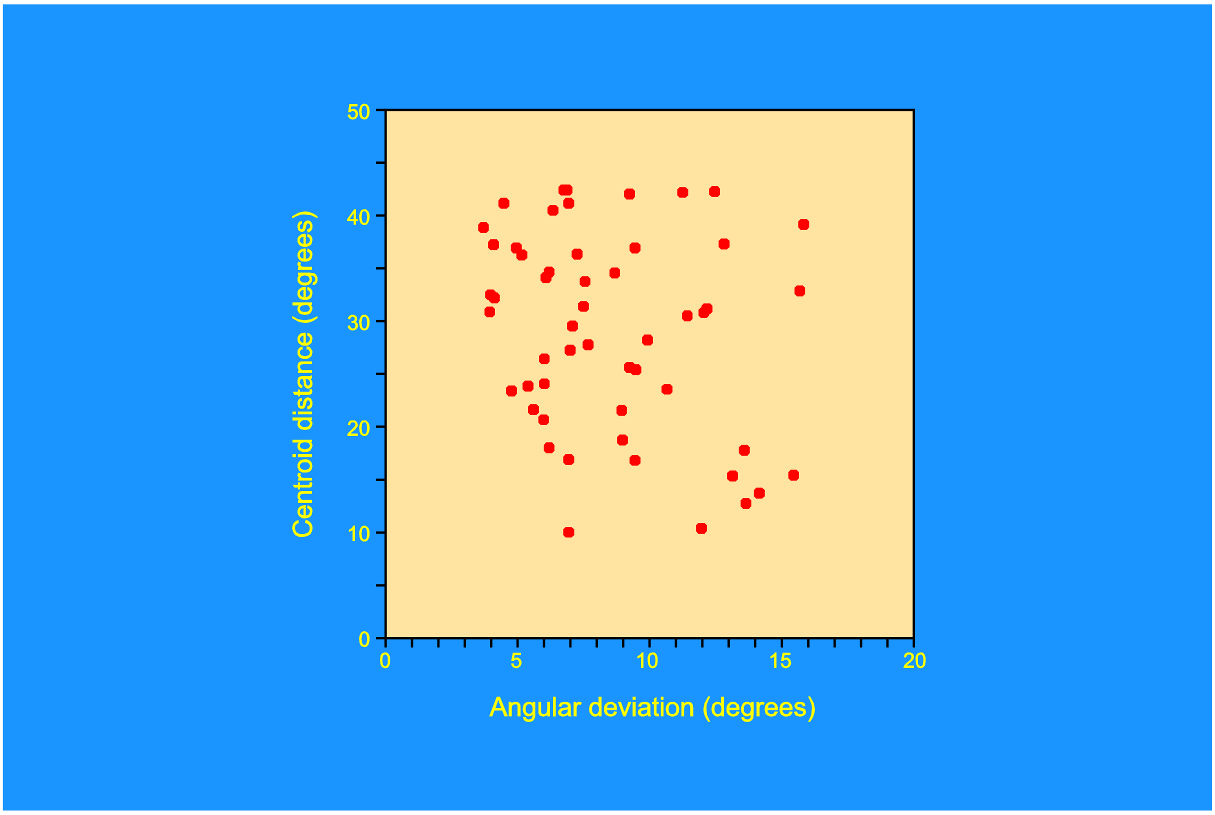

Figure 5. B) Scatter graph of angular deviation vs. centroid distance for a four-end-member model does not show any outliers. See text for discussion.

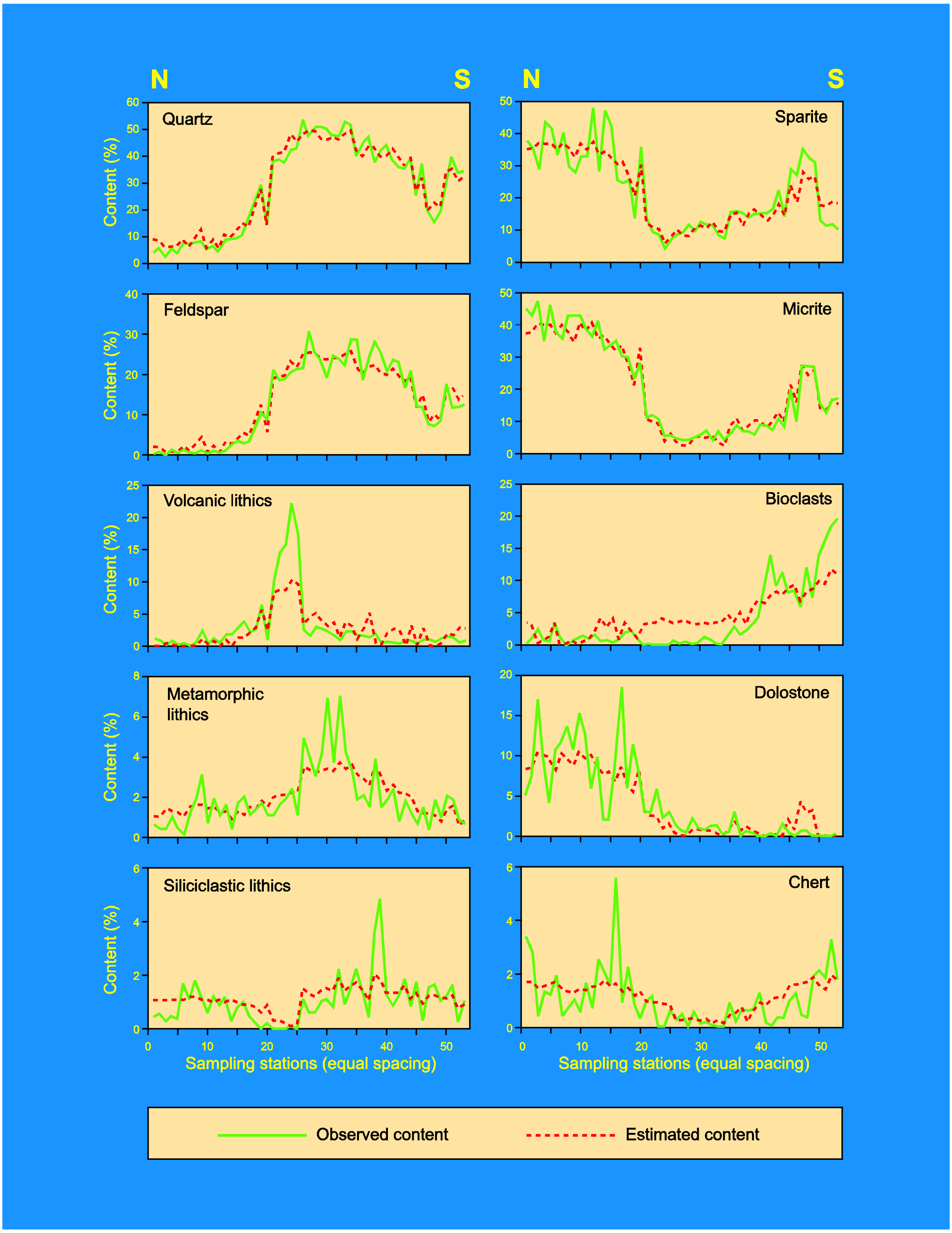

Figure 6: Comparison of observed and estimated alongshore variation of the ten grain types. Note smoothing effects due to reduced variance of the mixing model.

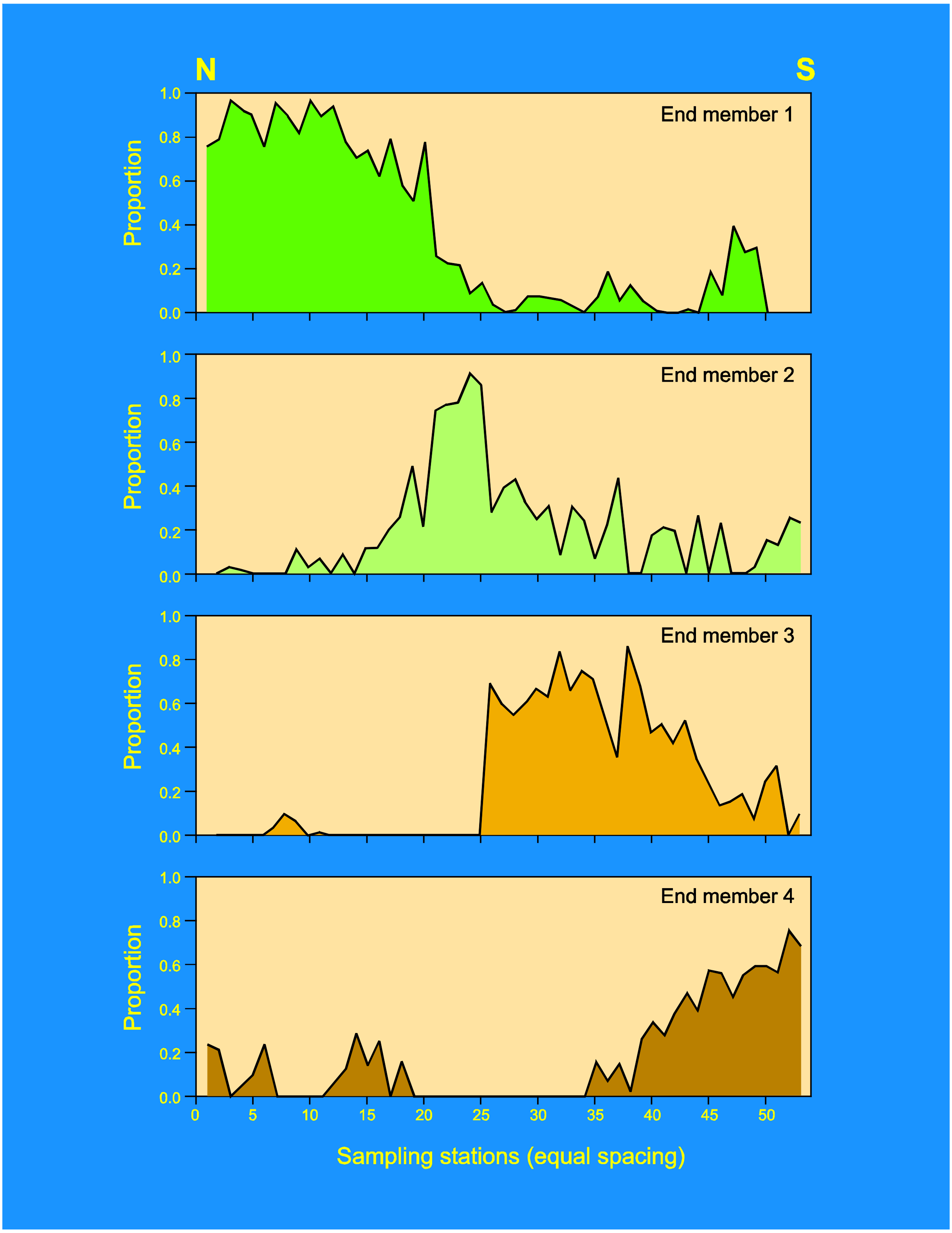

Figure 7: Estimated contributions of the modelled end members to the Northern Adriatic beach sands.

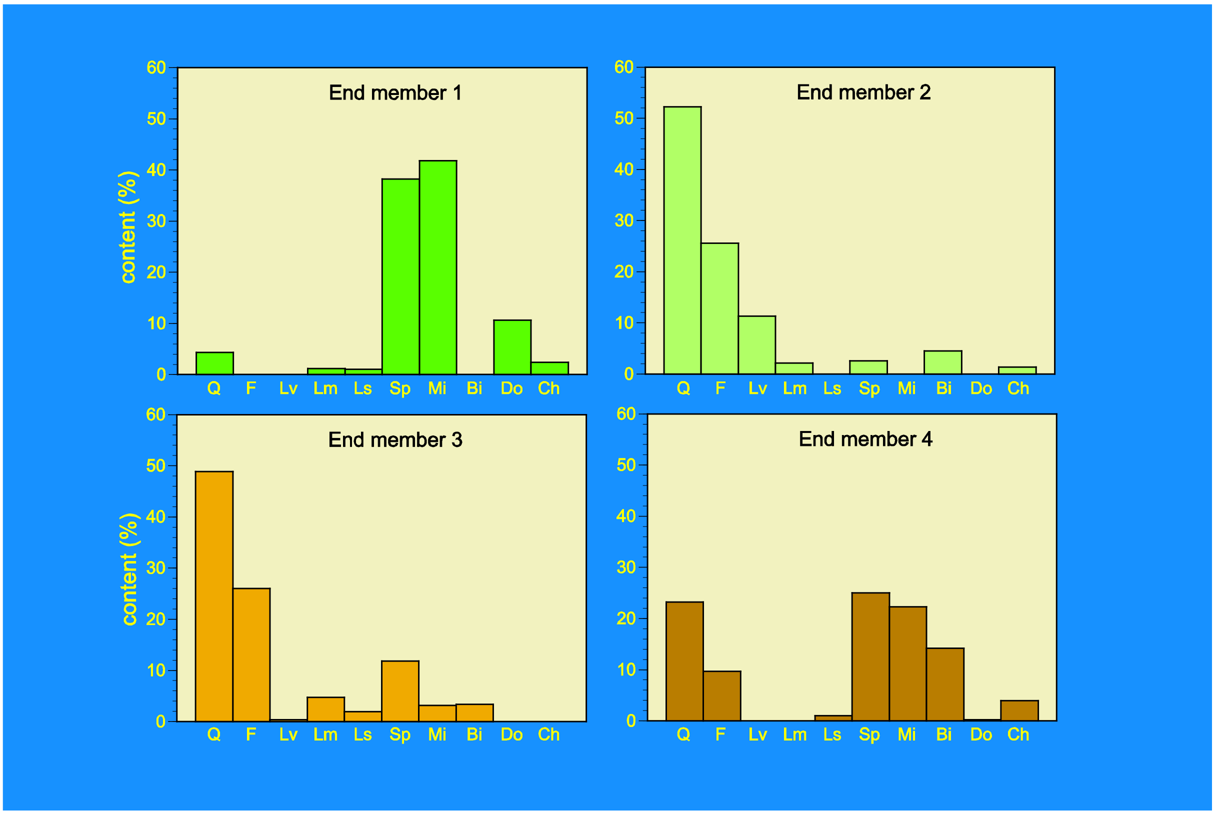

Figure 8: Composition of modelled end members of the Northern Adriatic beach sands.

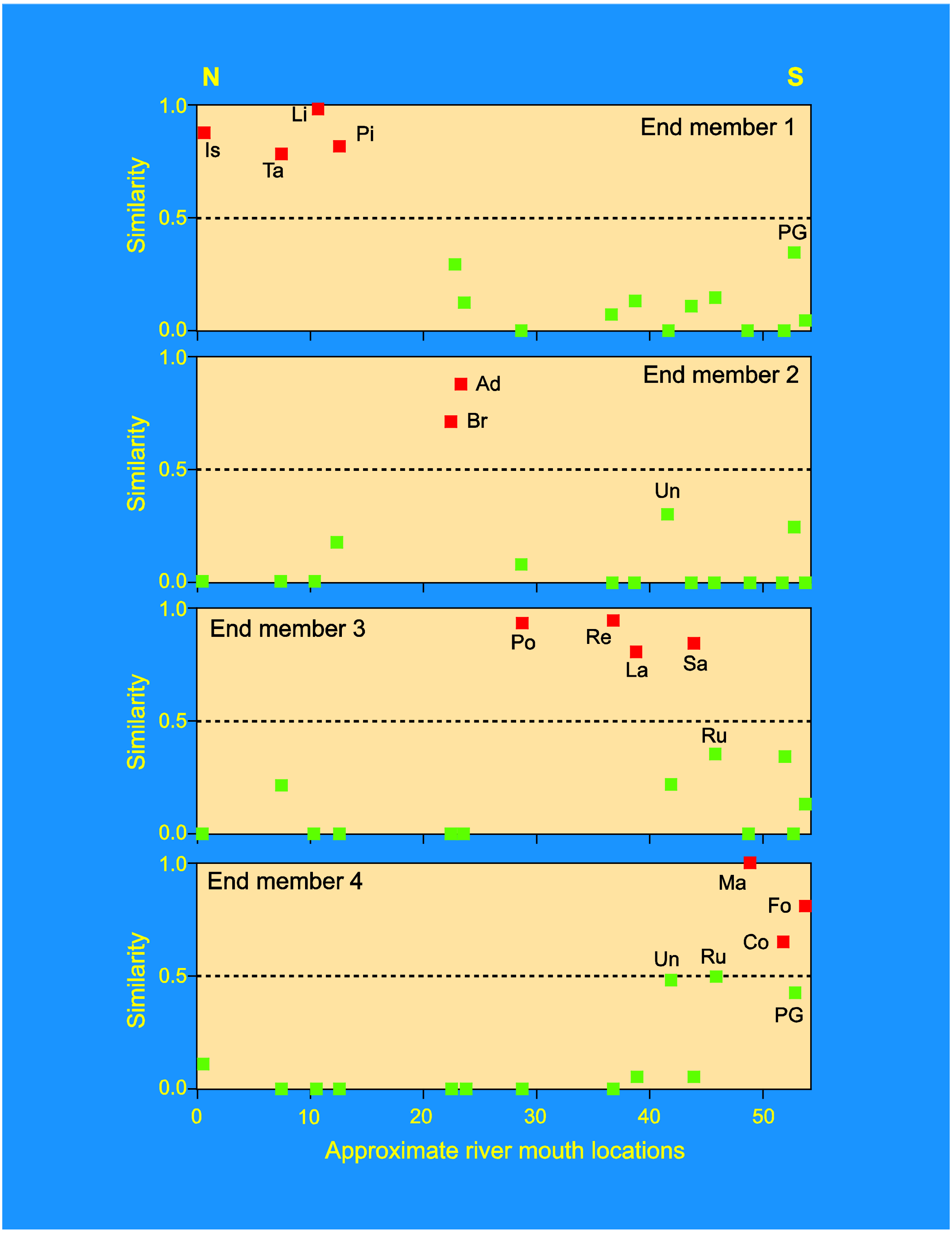

Figure 9: Similarity of candidate source assemblages to modelled end members (compare to Fig. 7). PG = Punta Gabicce; for other abbreviations see Table 4.

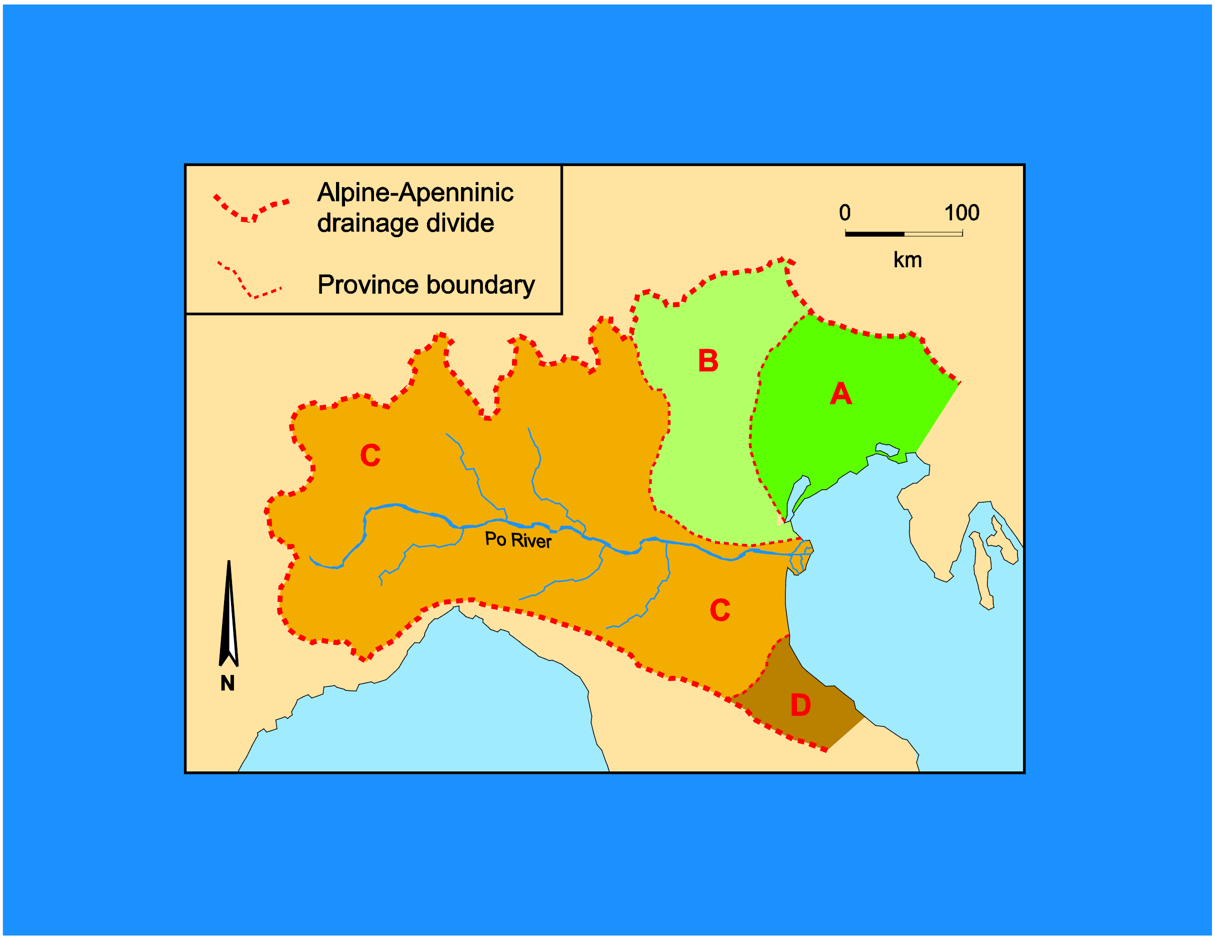

Figure 10: The four sedimentary-petrological provinces (cf. Edelman, 1933) distinguished on the basis of the modelling results.

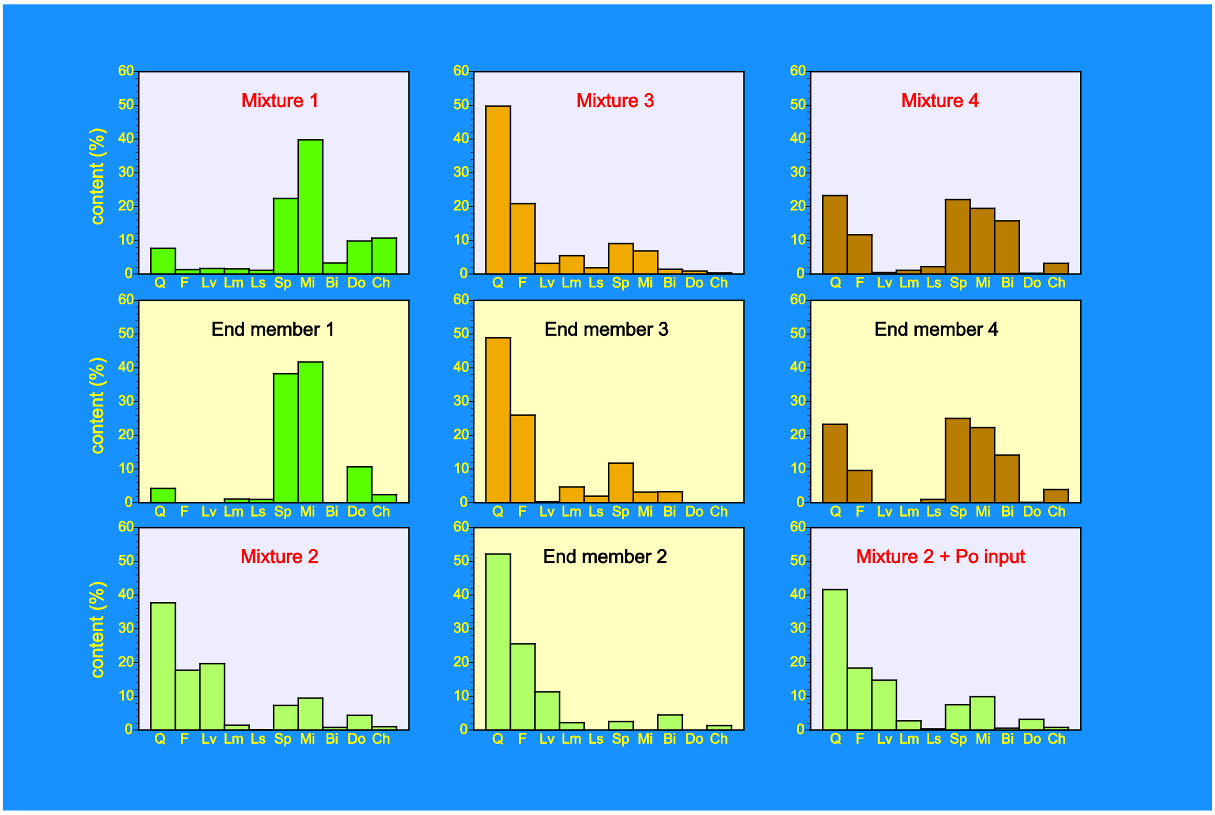

Figure 11: Synthetic weighted mean compositions of sediments supplied by each source area compared to the modelled end members. End member 2 cannot be well approximated by Brenta-Adige sediments; addition of Po sediments improves goodness-of-fit considerably (see text for full discussion of results).

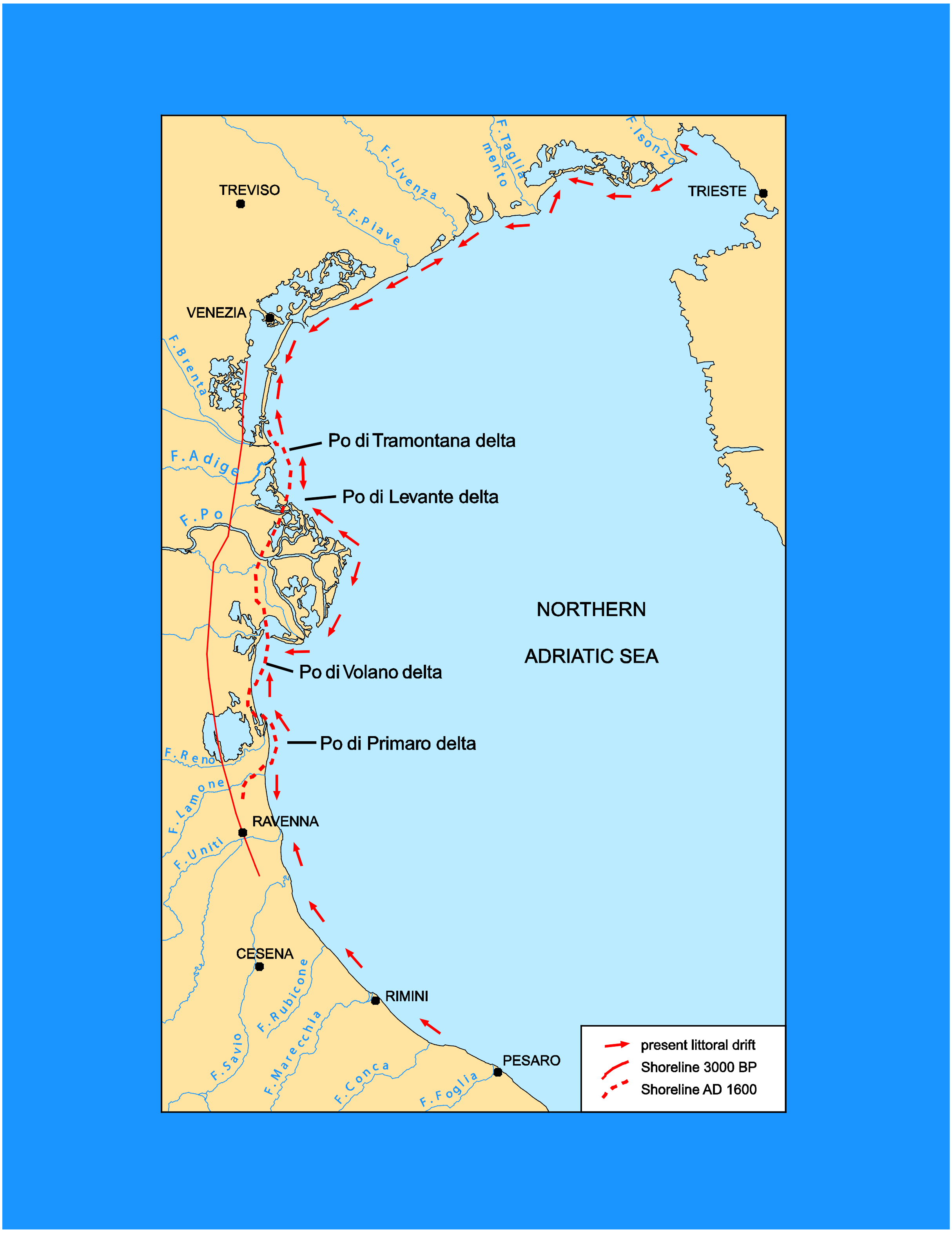

Figure 12: Present littoral drift (Bondesan et al., 1978; Brambati et al., 1978; Dal Cin, 1983), ancient shorelines and delta lobes (Nelson, 1970; Colantoni et al., 1979; Gandolfi et al.,1982).

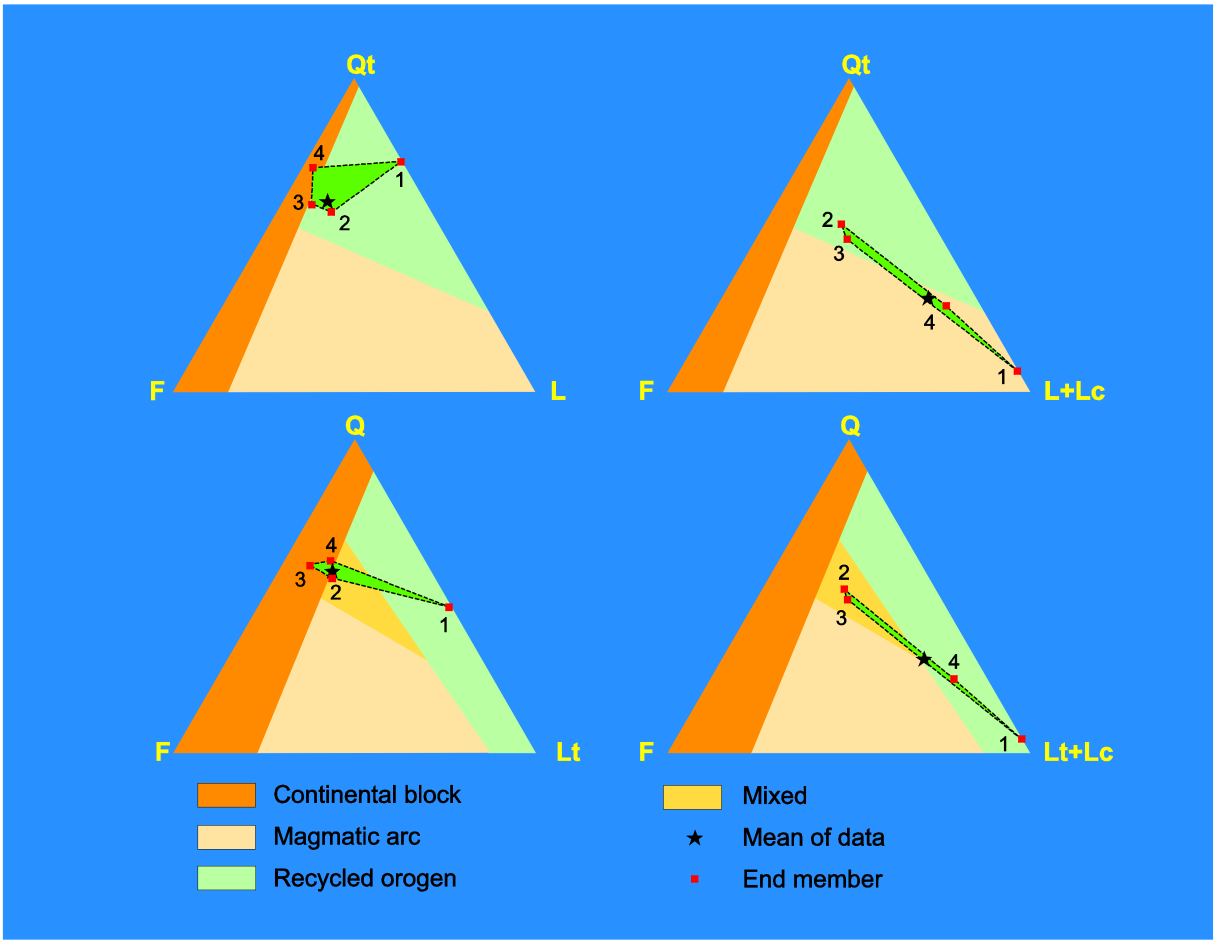

Figure 13: Ternary provenance diagrams (Dickinson, 1985), showing predictive regions of second-order provenance types for Adriatic beach sands. See text for discussion.